

NEMETH UNIFORM BRAILLE SYSTEM

Dr. Abraham Nemeth

MARCH 4, 2009

Scientific Uses

Compiled by Joyce H. Hull

CHAPTER 4

COMPUTATIONAL SCHEMES

CHAPTER 4 TOPICS

- 4.0 Chapter 4 Reference List
- 4.1 Recognizing a Computational Scheme
- 4.2 Format Requirements
- 4.3 Help for Young Children
 - 4.3.1 Column Separation
 - 4.3.2 Avoiding Paper Creep
- 4.4 Carried-Number Line in Addition
- 4.5 Cancellation When Subtracting
 - 4.5.1 Teacher Alert
- 4.6 Cancellation in General
- 4.7 The Abacus and the Calculator

4.0 Chapter 4 Reference List

When writing, locate a description of interest in the first column; then find its matching symbol in the second column.

When reading, locate a symbol of interest in the second column; then find its matching description in the first column.

Description	Symbol
begin-cancellation indicator	⠄⠄⠄⠄⠄⠄ (12346)
end-cancellation indicator	⠄⠄⠄⠄⠄⠄ (246)
first component of ruled line	⠄⠄⠄⠄⠄⠄ (5)
general component of ruled line	⠄⠄⠄⠄⠄⠄ (25)
one-character cancellation indicator	⠄⠄⠄⠄⠄⠄ (456)
single-cell representation of a two-digit number	
10	⠄⠄⠄⠄⠄⠄ (1356)
11	⠄⠄⠄⠄⠄⠄ (12)
12	⠄⠄⠄⠄⠄⠄ (123)
13	⠄⠄⠄⠄⠄⠄ (125)
14	⠄⠄⠄⠄⠄⠄ (1256)
15	⠄⠄⠄⠄⠄⠄ (126)
16	⠄⠄⠄⠄⠄⠄ (1235)
17	⠄⠄⠄⠄⠄⠄ (12356)
18	⠄⠄⠄⠄⠄⠄ (1236)

4.1 Recognizing a Computational Scheme

A computational scheme can be recognized by its two-dimensional disposition on the printed page and the vertical alignment of its elements into columns clearly intended for actual or potential computation. A fraction, although its terms may be vertically aligned, is not a computational scheme because the alignment is not for the purpose of computation. Thus, a computational scheme is displayed text which must be treated as spatial material. All the examples in this chapter are computational schemes.

4.2 Format Requirements

There must always be a blank line above and a blank line below every computational scheme. The transition to a new braille page before the scheme begins or after it ends does not nullify this requirement. However, the line of hyphens that marks the transition to a new print page does not require such blank lines.

The entire computational scheme must be confined to a single braille page by employing, if necessary, every space-capturing device permitted by NUBS.

Two or more computational schemes may be brailled across the page provided that there is a blank column, at least two cells wide, between adjacent computational schemes. In determining the placement of a computational scheme, the simulated ruled lines which it contains must be taken into account to abide by this two-cell separation requirement.

The label, if any, of a computational scheme may be a number, a letter, or some other symbol or combination of symbols. Such symbols may be enclosed, punctuated, or modified in some other way. In a computational scheme, the numeric indicator should be omitted when it otherwise would be required. The label of a computational scheme is not part of the scheme. Accordingly, if the label requires the numeric indicator according to some other rule, that indicator must be supplied.

If possible, a label must be placed to the left of the computational scheme alongside the first line of original data. A carried-number line in an addition scheme, replacements of cancelled numbers in a subtraction scheme, and the quotient in a division scheme are not original data. If the label can be so placed, there must be a blank column exactly one cell wide between the label and the scheme, and the simulated ruled lines in the scheme must be taken into account in determining the placement of the scheme relative to its label.

If the label cannot be placed alongside the scheme, it must be placed above the scheme, and separated from it by exactly one blank line.

Two or more labeled computational schemes may be brailled across the page provided the two-cell separation requirement is observed between adjacent schemes.

It is not necessary for a computational scheme to display the entire computation, including the result. In many textbook exercises, the entire scheme is displayed except for the result of the computation. In many illustrative examples, a computational scheme is built up step by step so that only partial computational schemes of increasing completeness are displayed at each step.

In a computational scheme for addition or subtraction, the sign of operation (plus or minus) should not be an obstruction in any column in which computation, actual or potential, is to be performed. The sign of operation should be moved to the left as far as is necessary to avoid becoming such an obstruction. In a multiplication scheme, it is not necessary to move the times sign out of the way, since the algorithm for multiplication does not involve moving up and down columns in which the multiplication sign might be present.

Since it is inconvenient and impractical to draw a ruled line in braille to correspond to a ruled line in print, a series of dots (25) serves to simulate a horizontal ruled line in braille, except for its first component which is always dot 5. We will use the expression "ruled line" rather than the longer expression "simulated ruled line" for convenience. In the 1972 Nemeth Code, a ruled line was called a "separation line."

In an addition or subtraction scheme, the length of the ruled line is determined globally. Its first component (dot 5) must fall in the column one cell to the left of the leftmost character of the entire computational scheme. Its last component (dots 25) must fall in the column one cell

places as may be required. In such cases, this method of separating columns of figures is suitable for adults as well as for children.

EXAMPLE (showing column separation)

$$\begin{array}{r}
 2.54 \\
 \times 16.7 \\
 \hline
 1778 \\
 1524 \\
 254 \\
 \hline
 42.418
 \end{array}$$

This is a fifth-grade multiplication example. Since there are two digits to the right of the decimal point in the multiplicand and one digit to the right of the decimal point in the multiplier, the decimal point has been placed in the product so that it is followed by three digits. This technique is not confined to multiplication schemes; it may be used in any computational scheme when appropriate.

4.3.2 Avoiding Paper Creep

People who use a braille in the computation of a long division example (strongly recommended) frequently experience the "creeping paper" problem. As the paper is rolled up to the quotient line to enter the latest digit or the latest algebraic term and then rolled back down to the current work area, there is a tendency for the paper to "creep" so that the new braille entry is too close to the previous braille entry for comfortable reading. With a repetition of this "roll up and roll down" activity as each new digit or term is developed, which is also time-consuming, this "creeping" behavior finally causes the new braille and the old braille to merge so that both are illegible. To solve this problem, it is suggested that each new digit or term, as it is developed, be entered against the left margin abreast of the partial product about to be generated. Thus, the paper is always advanced and never rolled back. When the computation is complete, the digits or terms of the quotient can be read from top to bottom of the paper against its left margin. The final remainder is also available on the last line of the computation. If desired, these can now be transferred to the quotient line. The student should be informed that this is a "braille-only" ploy to avoid "paper creep." If the work is to be handed to a sighted teacher who knows braille, or if it is to be turned into writing by a vision teacher for submission to a subject-matter teacher, the quotient line must be supplied. Notice the NUBS method for showing the format of a long-division problem.

4.4 Carried-Number Line in Addition

When imparting to young children the skill of adding columns of numbers, an intermediate phase in the development of that skill is to have the children write down the number that must be carried from one column to the neighboring column on the left, whenever such a carry is necessary. When the addition skill is well established, these numbers, of course, are processed mentally rather than written down.

In print, carried numbers are usually shown as reduced in size and appear directly above the top entry of the column to which they apply. In this System, we devise a "carried-number line." Each component of the "carried-number line," except the first, consists of dots 2356. The first component of the carried-number line consists of dots 56. Its length is the same as the length of the ruled line at the bottom of the addition scheme.

Children must be made aware of the fact that the "carried-number line" is a "braille-only device." The print method of showing carried numbers should be explained to them.

EXAMPLE (showing a carried-number line)

$$\begin{array}{r}
 11 \\
 368 \\
 \hline
 572 \\
 940
 \end{array}$$

Since the carried-number line is longer at both ends than the terms that participate in the addition example, and since its first component is different from the remaining components, the carried-number line cannot be confused with a string of 7's as a part of the example.

4.5 Cancellation When Subtracting

When imparting to young children the skill of subtracting one number from another, an intermediate phase in the development of that skill is to have the children actually write down the numbers that play a role in a procedure called "renaming" (sometimes called "borrowing"). The form of this renaming procedure involves the cancellation of some numbers and their replacement by others. When the subtraction skill is well established, the renaming process is carried out mentally. Until then, the children must pass through this intermediate stage.

In print, we can, with one stroke (no pun intended), simultaneously show the extent of the cancellation and preserve the readability of what has been cancelled. In braille, such a feat is not easily achieved.

$$\begin{array}{r}
 2 \quad \quad \quad \begin{array}{cccc} & 7 & 12 & 5 & 13 \\ & 8 & 2 & 6 & 3 \\ - & 2 & 8 & 3 & 6 \\ \hline & 5 & 4 & 2 & 7 \end{array}
 \end{array}$$

In this example, only one line of numbers above the minuend is required. If we number the columns 1 to 4 from right to left, we find that the compressed two-digit number 13 is in column 1 and the two-digit compressed number 12 is in column 3. We now can complete the problem by subtracting 6 from 13, 3 from 5, 8 from 12, and 2 from 7, entering each result in the remainder line. Since, after the renaming procedure, subtraction is possible in each column, we can even proceed to perform the individual subtractions from left to right, thereby avoiding backspacing when using the braillewriter.

$$\begin{array}{r}
 3 \quad \quad \quad \begin{array}{cccc} & 4 & 5 & 6 & 7 \\ - & 2 & 9 & 6 & 8 \\ \hline & & & & \end{array}
 \end{array}$$

$$\begin{array}{r}
 3a \quad \quad \quad \begin{array}{cccc} & 5 & 17 & & \\ & 4 & 5 & 6 & 7 \\ - & 2 & 9 & 6 & 8 \\ \hline & & & & \end{array}
 \end{array}$$

$$\begin{array}{r}
 3b \quad \quad \quad \begin{array}{cccc} & & 15 & 17 & \\ & 4 & 5 & 6 & 7 \\ - & 2 & 9 & 6 & 8 \\ \hline & & & & \end{array}
 \end{array}$$

$$\begin{array}{r}
 3c \quad \quad \quad \begin{array}{cccc} & 14 & 15 & & \\ & 3 & 4 & 5 & 17 \\ & 4 & 5 & 6 & 7 \\ - & 2 & 9 & 6 & 8 \\ \hline & 1 & 5 & 9 & 9 \end{array}
 \end{array}$$

Example 3 is a proposed subtraction scheme; the student's task is to compute the remainder after the subtraction.

Example 3a is a "snapshot" of the status of the computation after having processed the ones column. Whenever the subtraction cannot be performed immediately, a renaming process occurs. In this case, 8 cannot be immediately subtracted from 7; therefore, the 7 in the current column is cancelled and is replaced (above) by a number which is 10 larger than itself, namely, 17. Then, the 6 in the tens column is cancelled and replaced (above) by a number which is 1 less than itself, namely, 5. Note that 4 thousands plus 5 hundreds plus 5 tens plus

Example 1 shows how two fractions are multiplied. The 3 in the first numerator cancels the 9 in the second denominator. These cancelled numbers are replaced by 1 and 3 respectively. The number 8 in the first denominator and the 4 in the second numerator have a common factor of 4. These numbers are cancelled and replaced by the result obtained by dividing each one by 4. The result is obtained by multiplying the two new numerators and the two new denominators.

Example 2 shows the product of four algebraic expressions in which the third factor has been cancelled without a replacement.

4.7 The Abacus and the Calculator

The abacus and the talking calculator have become staples in the math curriculum of blind children. When they are properly used, they yield results with the speed and accuracy that matches or even surpasses the speed and accuracy attained by sighted people. Therefore, they are valuable aids when put into the hands of blind users.

But they also pose a clear danger to the development of math skills by children if they are introduced too early. Before learning to use the abacus or the calculator, a blind child must become comfortable with the pencil-and-paper algorithms for performing the basic arithmetic operations of addition, subtraction, multiplication and division. Without this background, he will be at a loss to understand a phrase like "put down the 3 and carry a 2" when performing addition. He will have no clear conception of the process of borrowing or renaming when performing subtraction. He will have no experience with the "stairstep" pattern produced by the partial products when performing multiplication. He will have no concept of the phrase "bring down the next figure" when performing long division. All of these ideas will arise as the blind child acquires the pencil-and-paper skills that his sighted peers learn.

The ideal tool for teaching these skills is the Perkins braille. With this braille, alignment of figures is easily achieved. The two-dimensional layout of the algorithm is immediately apparent. A permanent record on paper is produced for later review.

When the pencil-and-paper skills are acquired in the performance of arithmetic operations, they are easily transferred to analogous algebraic operations. The abacus and the calculator are restricted to arithmetic operations only; they are useless in the performance of algebraic operations.

Therefore, access to the abacus and the calculator should be deferred until after the blind child has acquired the skills described above. He will then have both the skill and the knowledge of the algorithms underlying the basic arithmetic operations and the means for achieving the speed and accuracy required in everyday activities.

CHAPTER 5 EXTENSION TO MATHEMATICS

Mathematics is part of the natural sciences. These encompass the fields of mathematics, physics, chemistry, and the various branches in each of these fields. They exclude the life sciences and the humanities. There are no rigid boundaries among these fields, so that the notation in any of these fields will find its way into some other field. For example, the notational constructs in organic chemistry are frequently found in the literature that deals with microbiology. The symbols of mathematics are prominent in the field of physical chemistry.

This chapter concentrates on the extension to mathematics. We turn our attention to the field of mathematics first because its notation, to a large extent, permeates all of the other natural sciences.

In Chapter 3, we proposed a whole set of symbols that are not only mathematical, but which have found their way into the general literature. We also undertook a cursory analysis of fractions and radicals, and we addressed the basics of subscripts, superscripts and related matters. There are, however, other symbols that are found almost exclusively in the field of mathematics and its various branches. One of our guiding principles in Chapter 1, (principle h) requires that notation be represented without regard to its meaning. Therefore, if a symbol commonly found in one branch of mathematics happens to appear in another branch, or even in another field, but with a different meaning, this will have no effect on how it is represented in braille.

First, we include some symbols that were deliberately left out in earlier chapters. In those chapters we addressed the kind of text likely to be found in everyday reading material. Therefore, we presented only the basics in each covered category. Here we supply more details for each of those categories. We then go on to categories of a more technical nature which were not even mentioned in earlier chapters.

CHAPTER 5 TOPICS

5.0 Mathematics Symbol Set

- 5.0.1 Arrows (continued)
- 5.0.2 ASCII Character Set
 - 5.0.2.1 Auxiliary Notation
- 5.0.3 Calculus and Analysis
- 5.0.4 Comparison Signs (continued)
- 5.0.5 Computer and Calculator
- 5.0.6 Fractions and Radicals (continued)
- 5.0.7 Functions and Their Abbreviations
- 5.0.8 Geometry
 - 5.0.8.1 Geometric Modifiers
 - 5.0.8.2 Geometric Shapes
- 5.0.9 Grouping Signs (continued)
- 5.0.10 Indicators (continued)
- 5.0.11 Logic and Set Theory

- 5.0.12 Modifiers
- 5.0.13 Operation Signs (continued)
- 5.0.14 Punctuation (continued)
- 5.0.15 Special Letters (continued)
 - 5.0.15.1 Crossed
 - 5.0.15.2 Global Operation Signs
 - 5.0.15.3 Hebrew
 - 5.0.15.4 Miscellaneous
 - 5.0.15.5 Number Sets
 - 5.0.15.6 Obsolete Greek
 - 5.0.15.7 Variant Greek
- 5.0.16 Trigonometry
- 5.1 Arrows (continued)
- 5.2 The ASCII Character Set
 - 5.2.1 Auxiliary Notation
- 5.3 Calculus and Analysis
- 5.4 Comparison Signs (continued)
- 5.5 Computer and Calculator
- 5.6 Delimiters (continued)
- 5.7 Fractions and Radicals (continued)
 - 5.7.1 Fractions of Higher Order
 - 5.7.2 Spatial Representation of Fractions
 - 5.7.3 Continued Fractions
 - 5.7.4 Indexed Radicals
 - 5.7.5 Radicals of Higher Order
 - 5.7.6 Spatial Representation of Radicals
 - 5.7.7 Fractions and Radicals Together in a Spatial Representation
- 5.8 Functions and Their Abbreviations
 - 5.8.1 Spacing with Function Abbreviations
- 5.9 Geometry
 - 5.9.1 Geometric Modifiers
 - 5.9.2 Geometric Shapes
- 5.10 Grouping Signs (continued)
- 5.11 Logic and Set Theory
- 5.12 Special Letters (continued)
 - 5.12.1 Crossed
 - 5.12.2 Global Operation Signs
 - 5.12.3 Hebrew
 - 5.12.4 Miscellaneous
 - 5.12.5 Number Sets
 - 5.12.6 Obsolete Greek
 - 5.12.7 Variant Greek
- 5.13 Subscripts and Superscripts (continued), and Modifiers
 - 5.13.1 Primes Together with Indices
 - 5.13.2 Simultaneous and Non-Simultaneous Indices
 - 5.13.3 Left Indices
 - 5.13.4 Numeric Subscripts

5.13.5 Higher-Order Indices

5.13.6 Overscripts and Underscripts

5.13.7 Modifiers

5.14 Trigonometry

5.0 Mathematics Symbol Set

This Mathematics Symbol Set contains symbols found in texts that are more technical. Symbols that are likely to be found in everyday texts are listed in the Basic Literary Symbol Set. The mathematics symbols listed below have been sorted into categories and the categories have been alphabetized.

When writing, locate a description of interest in the first column; then find its matching symbol in the second column for braille or in the last column for print.

When reading, locate a symbol of interest in the second column for braille or the last column for print; then find its matching description in the first column.

The absence of a tactile graphic in the last column, except in the case of function abbreviations, signifies an indicator.

Description	Braille	Dots	Graph
5.0.1 ARROWS (continued)			
northeast	⠠⠠⠠	(4,45,135)	
northwest	⠠⠠⠠	(4,45,246)	
northwest-southeast	⠠⠠⠠⠠	(4,56,123456)	
southeast	⠠⠠⠠	(4,56,135)	
southwest	⠠⠠⠠	(4,56,246)	
southwest-northeast	⠠⠠⠠⠠	(4,45,123456)	
5.0.2 ASCII CHARACTER SET (excludes letters and digits)			
ampersand	⠠⠠	(4,12346)	&
apostrophe	⠠	(3)	'
asterisk	⠠⠠	(456,246)	*
at sign	⠠⠠	(4,1)	@
backslash	⠠⠠	(456,16)	\
caret	⠠⠠	(6,26)	^
colon	⠠	(25)	:
comma	⠠	(16)	,
crosshatch	⠠⠠⠠	(46,3456)	#
dollar sign	⠠⠠	(4,234)	\$
equals	⠠⠠	(123456)	=
exclamation	⠠	(235)	!
grave accent	⠠⠠	(4,26)	`
hyphen	⠠	(36)	-
left angle bracket	⠠⠠	(4,126)	<
left brace	⠠⠠⠠	(46,12356)	{
left bracket	⠠⠠	(4,12356)	[

left parenthesis	⠠	(12356)	(
percent sign	⠠⠠	(46,356)	%
period	⠠	(12456)	.
plus	⠠	(346)	+
question mark	⠠	(236)	?
quote	⠠⠠	(6,2356)	“
right angle bracket	⠠⠠	(4,345)	>
right brace	⠠⠠	(46,23456)	}
right bracket	⠠⠠	(4,23456)]
right parenthesis	⠠	(23456))
semicolon	⠠	(23)	;
slash	⠠⠠	(456,34)	/
space		(no dots)	
tilde	⠠⠠	(46,35)	~
underscore	⠠⠠	(6,36)	_
vertical bar	⠠⠠	(4,1256)	

5.0.2.1 AUXILIARY NOTATION

begin upper-case passage	⠠⠠⠠⠠	(6,6,56)	
end upper-case passage	⠠⠠	(6,3)	
upper case, one letter	⠠	(6)	
upper case, one word	⠠⠠	(6,6)	

5.0.3 CALCULUS AND ANALYSIS

absolute value	⠠⠠	(4,1256)	
del	⠠⠠	(456,245)	∇
double vertical bar	⠠⠠⠠	(4,4,1256)	
infinity	⠠⠠	(4,236)	∞
integral	⠠	(2346)	∫
large boldface pi	⠠⠠⠠	(456,6,1234)	π
large boldface sigma	⠠⠠⠠	(456,6,234)	Σ
limit	⠠⠠⠠		lim
logarithm	⠠⠠⠠		log
lower limit	⠠⠠⠠⠠⠠		<u>lim</u>
nabla	⠠⠠	(456,245)	∇
natural logarithm	⠠⠠		ln
norm	⠠⠠⠠	(4,4,1256)	
partial derivative	⠠⠠	(4,1246)	∂
prime	⠠	(3)	'
upper limit	⠠⠠⠠⠠⠠		<u>lim</u>
vertical bar	⠠⠠	(4,1256)	

5.0.4 COMPARISON SIGNS (continued)

approximately equal	⠠⠠	(456,35)	≈
contains the element	⠠⠠⠠	(4,4,24)	

contains the subset	⋮ ⋮ ⋮	(4,4,13456)	\supset	
does not contain the element	⋮ ⋮ ⋮	(46,4,24)		\notin
does not include	⋮ ⋮ ⋮	(46,4,13456)	$\not\supset$	
greater than or equal	⋮ ⋮	(456,345)	\geq	
implies	⋮ ⋮ ⋮	(4,4,13456)	\supset	
includes	⋮ ⋮ ⋮	(4,4,13456)	\supset	
is an element	⋮ ⋮ ⋮	(4,4,15)	\in	
is a subset	⋮ ⋮ ⋮	(4,4,12346)	\subseteq	
is not an element	⋮ ⋮ ⋮	(46,4,15)	\notin	
is not a subset	⋮ ⋮ ⋮	(46,4,12346)	$\not\subseteq$	
less than or equal	⋮ ⋮	(456,126)	\leq	
not equal to	⋮ ⋮	(46,123456)	\neq	
not greater than	⋮ ⋮	(46,345)	$\not\geq$	
not less than	⋮ ⋮	(46,126)	$\not\leq$	

5.0.5 COMPUTER AND CALCULATOR

begin keypad legend	⋮	(246)
begin screen display	⋮	(12346)
runover indicator	⋮	(456)

5.0.6 FRACTIONS AND RADICALS (continued)

complexity indicator for fractions and radicals	⋮	(6)	
component of a radical ascender	⋮	(456)	
component of spatial fraction bar	⋮	(25)	
component of a spatial vinculum	⋮	(25)	
last component of radical ascender	⋮	(56)	
radical sign	⋮	(345)	$\sqrt{\quad}$
radical sign without vinculum	⋮ ⋮ ⋮	(46,46,345)	$\sqrt{\quad}$

5.0.7 FUNCTIONS AND THEIR ABBREVIATIONS

amplitude	amp
antilogarithm	antilog
arc	arc
argument	arg
cologarithm	colog
cosine + <i>isin</i>	cis
coversine	covers
curl	curl
determinant	det
dimension	dim
divergence	div
error function	erf
exponential	exp
exsecant	exsec

gradient	grad
haversine	hav
imaginary	im
infimum	inf
limit	lim
logarithm	log
lower limit	<u>lim</u> ⋮ ⋮ ⋮ ⋮
maximum	max
minimum	min
modulo	mod
modulus	mod
natural logarithm	ln
rank	rnk
real	re
supremum	<u>sup</u>
upper limit	<u>lim</u> ⋮ ⋮ ⋮ ⋮
versine	vers

5.0.8 GEOMETRY

5.0.8.1 GEOMETRIC MODIFIERS

full line	⋮ ⋮	(4,123456)	←—→
half lines			
closed left	⋮ ⋮ ⋮	(456,4,135)	●—→
closed right	⋮ ⋮ ⋮	(456,4,246)	←—●
open left	⋮ ⋮ ⋮	(46,4,135)	○—→
open right	⋮ ⋮ ⋮	(46,4,246)	←—○
line segments			
left closed, right closed	⋮ ⋮ ⋮	(456,456,156)	●—●
left closed, right open	⋮ ⋮ ⋮	(456,46,156)	●—○
left open, right closed	⋮ ⋮ ⋮	(46,456,156)	○—●
left open, right open	⋮ ⋮ ⋮	(46,46,156)	○—○

5.0.8.2 GEOMETRIC SHAPES

angle	⋮ ⋮ ⋮	(456,5,2346)	∠
arc, concave	⋮ ⋮ ⋮	(456,5,1)	∪
arc, convex	⋮ ⋮ ⋮	(456,5,3)	∩
circle	⋮ ⋮ ⋮	(456,5,14)	○
diamond	⋮ ⋮ ⋮	(456,5,145)	
ellipse	⋮ ⋮ ⋮	(456,5,15)	
equilateral triangle	⋮ ⋮ ⋮	(456,5,25)	
hexagon	⋮ ⋮ ⋮	(456,5,235)	
is parallel	⋮ ⋮ ⋮	(456,5,123)	
is perpendicular	⋮ ⋮ ⋮	(456,5,1234)	

octagon	⠠⠠⠠⠠	(456,5,236)
oval	⠠⠠⠠⠠	(456,5,135)
parallelogram	⠠⠠⠠⠠	(456,5,1245)
pentagon	⠠⠠⠠⠠	(456,5,26)
quadrilateral	⠠⠠⠠⠠	(456,5,12345)
rectangle	⠠⠠⠠⠠	(456,5,1235)
rhombus	⠠⠠⠠⠠	(456,5,125)
right angle	⠠⠠⠠⠠	(456,5,1456)
right triangle	⠠⠠⠠⠠	(456,5,1236)
square	⠠⠠⠠⠠	(456,5,256)
star	⠠⠠⠠⠠	(456,5,234)
trapezoid	⠠⠠⠠⠠	(456,5,1356)
triangle	⠠⠠⠠⠠	(456,5,2345)

5.0.9 GROUPING SIGNS (continued)

barred signs

braces

left	⠠⠠⠠⠠	(456,46,12356)	{
right	⠠⠠⠠⠠	(456,46,23456)	}

brackets

left	⠠⠠⠠⠠	(456,4,12356)	[
right	⠠⠠⠠⠠	(456,4,23456)]

boldface signs

brackets

left	⠠⠠⠠⠠	(4,456,12356)	[
right	⠠⠠⠠⠠	(4,456,23456)]

vertical bar

left and right	⠠⠠⠠⠠	(4,456,1256)	
----------------	------	--------------	--

double vertical bars

left and right	⠠⠠⠠⠠	(4,4,1256)	
----------------	------	------------	--

extended signs

braces

left	⠠⠠⠠⠠	(46,6,12356)
right	⠠⠠⠠⠠	(46,6,23456)

brackets

left	⠠⠠⠠⠠	(4,6,12356)
right	⠠⠠⠠⠠	(4,6,23456)

parentheses

right	⠠⠠⠠⠠	(6,12356)
left	⠠⠠⠠⠠	(6,23456)

transcriber's enclosures

left	⠠⠠⠠⠠	(46,6,235)
------	------	------------

right vertical bar	$\cdot\cdot\cdot\cdot$	(46,6,256)
left and right half-brackets	$\cdot\cdot\cdot\cdot$	(4,6,1256)
lower left	$\cdot\cdot\cdot\cdot$	(4,56,12356)
lower right	$\cdot\cdot\cdot\cdot$	(4,56,23456)
upper left	$\cdot\cdot\cdot\cdot$	(4,45,12356)
upper right	$\cdot\cdot\cdot\cdot$	(4,45,23456)
vertical bar left and right	$\cdot\cdot\cdot\cdot$	(4,1256)

5.0.10 INDICATORS (continued)

begin keytop legend	$\cdot\cdot$	(246)
begin screen display	$\cdot\cdot$	(12346)
cancellation		
begin	$\cdot\cdot$	(12346)
end	$\cdot\cdot$	(246)
one character	$\cdot\cdot$	(456)
complexity indicator for fractions and radicals	$\cdot\cdot$	(6)
no boundary	$\cdot\cdot$	(4)
overscript	$\cdot\cdot\cdot\cdot$	(45,12346)
overstrike	$\cdot\cdot\cdot\cdot$	(5,12346)
runover	$\cdot\cdot$	(456)
shaded shape	$\cdot\cdot$	(46)
solid shape	$\cdot\cdot$	(456)
underscript	$\cdot\cdot\cdot\cdot$	(56,12346)

5.0.11 LOGIC AND SET THEORY

Cartesian product	$\cdot\cdot\cdot\cdot$	(46,246)	'
contains the element	$\cdot\cdot\cdot\cdot$	(4,4,24)	\in
contains the subset	$\cdot\cdot\cdot\cdot$	(4,4,13456)	\supset
does not contain the element	$\cdot\cdot\cdot\cdot$	(46,4,24)	\notin
does not contain the subset	$\cdot\cdot\cdot\cdot$	(46,4,13456)	$\not\supset$
empty set	$\cdot\cdot\cdot\cdot$	(456,356)	\emptyset
for all	$\cdot\cdot\cdot\cdot$	(456,456,3)	\forall
global Cartesian product	$\cdot\cdot\cdot\cdot$	(456,46,246)	'
global intersection	$\cdot\cdot\cdot\cdot$	(456,4,134)	$\dot{\cap}$
global product	$\cdot\cdot\cdot\cdot$	(456,6,1234)	
global sum	$\cdot\cdot\cdot\cdot$	(456,6,234)	
global union	$\cdot\cdot\cdot\cdot$	(456,4,136)	$\dot{\cup}$
if and only if	$\cdot\cdot\cdot\cdot$	(4,123456)	iff
implies	$\cdot\cdot\cdot\cdot$	(4,4,13456)	\supset
intersection	$\cdot\cdot\cdot\cdot$	(4,134)	\cap
is an element of	$\cdot\cdot\cdot\cdot$	(4,4,15)	\in
is implied by	$\cdot\cdot\cdot\cdot$	(4,4,12346)	\supset

is not an element of	⠠⠠⠠⠠	(46,4,15)	∉
is not a subset of	⠠⠠⠠⠠	(46,4,12346)	⊄
is a proper subset of	⠠⠠⠠⠠	(456,4,12346)	⊂
is a subset of	⠠⠠⠠⠠	(4,4,12346)	⊆
join	⠠⠠⠠	(4,1236)	∧
meet	⠠⠠⠠	(4,1234)	∨
there exists	⠠⠠⠠⠠	(456,456,26)	∃
there exists uniquely	⠠⠠⠠⠠	(456,46,26)	∃
union	⠠⠠⠠	(4,136)	∪

5.0.12 MODIFIERS

acute accent	⠠⠠	(4,35)	'
asterisk	⠠⠠	(456,246)	*
breve	⠠⠠⠠	(456,5,1)	
circumflex	⠠⠠	(6,26)	∧
diaeresis	⠠⠠	(46,25)	¨
dot	⠠⠠	(12456)	.
grave	⠠⠠	(4,26)	`
hat	⠠⠠	(6,26)	∧
left arrow	⠠⠠	(4,246)	←
macron	⠠⠠	(156)	-
right arrow	⠠⠠	(4,135)	→
ring	⠠⠠	(46,346)	°
tilde	⠠⠠	(46,35)	~
two-way arrow	⠠⠠	(4,123456)	↔
umlaut	⠠⠠	(46,25)	¨

5.0.13 OPERATION SIGNS (continued)

Cartesian product	⠠⠠	(46,246)	
global Cartesian product	⠠⠠⠠	(456,46,246)	
global intersection	⠠⠠⠠	(456,4,134)	
global union	⠠⠠⠠	(456,4,136)	
join	⠠⠠⠠	(4,1236)	
meet	⠠⠠⠠	(4,1234)	
minus or plus	⠠⠠	(456,36)	
plus or minus	⠠⠠	(456,346)	
union	⠠⠠⠠	(4,136)	

5.0.14 PUNCTUATION (continued)

ellipsis, vertical	⠠⠠	(456,2)	
--------------------	----	---------	--

5.0.15 SPECIAL LETTERS

5.0.15.1 CROSSED

b	⠠⠨	(4,12)
d	⠠⠨⠠	(4,145)
h	⠠⠨⠠	(4,125)
λ (lambda)	⠠⠨⠠⠠	(46,5,123)

5.0.15.2 GLOBAL OPERATION SIGNS

global product	⠠⠨⠠⠠	(456,6,1234)	Π
global sum	⠠⠨⠠⠠	(456,6,234)	Σ

5.0.15.3 HEBREW

aleph	⠠⠨	(46,2)	\aleph
beth	⠠⠨⠠	(46,23)	\beth

5.0.15.4 MISCELLANEOUS

partial derivative	⠠⠨⠠	(4,1246)	∂
Weierstrass p	⠠⠨⠠⠠	(46,5,1234)	

5.0.15.5 NUMBER SETS

complex	⠠⠨⠠⠠	(456,456,14)
integers	⠠⠨⠠⠠	(456,456,24)
natural	⠠⠨⠠⠠	(456,456,1345)
rational	⠠⠨⠠⠠	(456,456,1356)
real	⠠⠨⠠⠠	(456,456,1235)

5.0.15.6 OBSOLETE GREEK

koph or koppa	⠠⠨⠠⠠	(46,5,12345)
sampi	⠠⠨⠠⠠	(46,5,156)

5.0.15.7 VARIANT GREEK

alpha	⠠⠨⠠⠠	(46,5,1)
beta	⠠⠨⠠⠠	(46,5,12)
theta	⠠⠨⠠⠠	(46,5,1456)
sigma	⠠⠨⠠⠠	(46,5,234)
phi	⠠⠨⠠⠠	(46,5,124)

5.0.16 TRIGONOMETRY

arcsecant	arccsc
arccosine	arccos
arccotangent	arccot
arccotangent	arcctn
arcsecant	arcsec

arcsine	arcsin
arctangent	arctan
cosecant	csc
cosine	cos
cotangent	cot
cotangent	ctn
cosine	cos
cotangent	cot
cotangent	ctn
hyperbolic cosecant	csch
hyperbolic cosine	cosh
hyperbolic cotangent	coth
hyperbolic cotangent	ctnh
hyperbolic secant	sech
hyperbolic sine	sinh
hyperbolic tangent	tanh
inverse cosecant	csc ⁻¹
inverse cosine	cos ⁻¹
inverse cotangent	cot ⁻¹
inverse cotangent	ctn ⁻¹
inverse hyperbolic cosecant	csch ⁻¹
inverse hyperbolic cosine	cosh ⁻¹
inverse hyperbolic cotangent	coth ⁻¹
inverse hyperbolic cotangent	ctnh ⁻¹
inverse hyperbolic secant	sech ⁻¹
inverse hyperbolic sine	sinh ⁻¹
inverse hyperbolic tangent	tanh ⁻¹
inverse secant	sec ⁻¹
inverse sine	sin ⁻¹
inverse tangent	tan ⁻¹
radian	rad
secant	sec
sine	sin
tangent	tan

5.1 Arrows (continued)

In Chapter 3 we showed and discussed only horizontal and vertical arrows. There are, however, oblique arrows which are found mostly in chemistry, and these have been listed in Section 5.0.1 of the Mathematics Symbol Set.

Arrows are also used as modifiers in vector notation. NUBS permits these arrows to be omitted if there is other evidence that the notation represents a vector. However, omission of such arrows is optional--not mandatory.

5.2 The ASCII Character Set

NUBS does not require a separate set of symbols for computer or calculator notation. The current Computer Braille Code was devised with the underlying assumption that every character in the basic ASCII Character Set should be represented by a one-cell braille character. Since there are 95 printable characters in the basic ASCII Character Set but only 63 (64 if the space is counted) distinct braille characters in our 6-dot braille system, this goal of one-to-one correspondence between ASCII characters and braille characters is, *prima facie*, impossible to achieve. NUBS maintains that a strict one-to-one correspondence is unnecessary and proceeds accordingly.

The basic ASCII Character Set, excluding lower-case and upper-case letters and excluding digits, is shown together with their corresponding tactile graphics in Section 5.0.2 of the Mathematics Symbol Set.

Here is an example of a computer program in C++ to show how NUBS handles such a program. It is a binary search program to decide whether a particular value, x , occurs in an array v which contains n elements arranged in ascending order and numbered from 0 to $n-1$. The function *binary* returns a number between 0 and $n-1$ if a match is found between x and an element of v . This number represents the position in v where the match was found. If no match is found, the function *binary* returns a value of -1.

EXAMPLE (of a C++ computer program)

```
binary(x, v, n) /*find the position in v that matches x*/
int x, v[ ], n;
{int low, high, mid;
  low = 0;
  high = n-1;
  while (low <= high)
  {mid = (low+high)/2;
   if (x < v[mid])
     high = mid-1;
   else if (x > v[mid])
     low = mid+1;
   else /*found match*/
     return(mid);
  }return(-1);
}
```

With this implementation, several braille characters remain unassigned to ASCII characters. These are:

```
⠠⠠⠠⠠⠠⠠⠠⠠⠠
⠠⠠⠠⠠⠠⠠⠠⠠⠠
```

5.2.1 Auxiliary Notation

To assert that a single letter is upper case, use ⠠ (6) before that letter. This braille character is not assigned to any ASCII character, nor is it a prefix to any root. It is equivalent to holding down the shift key on a keyboard, entering the letter, and then releasing the shift key. This prefix may be used only before letters; it must not be used before any non-letter the way the shift key is used on a typewriter or on a computer keyboard.

The compound prefix ⠠⠠ (6,6) may be used to capitalize an entire word. If the word contains non-letters, it does not change the meaning of those non-letters. It resumes its effectiveness with the next letter after a non-letter symbol. Its effectiveness is terminated upon encountering the next delimiter. It is equivalent to setting Caps Lock to the On position on a keyboard and leaving it in that mode until the word has been completed, then reverting to the Caps Lock Off position.

The compound prefix ⠠⠠⠠ (6,6,56) may be used to introduce an upper-case passage. It does not change the meaning of non-letters within that passage. Its effect is cancelled by the ⠠⠠ (6,3) indicator. It is equivalent to setting Caps Lock to the On position on a keyboard, then setting it to the Off position upon encountering the ⠠⠠ (6,3) indicator.

Font switching in computer programs, either for emphasis or for any other purpose, is accomplished in the same way as it is accomplished in NUBS at large.

5.3 Calculus and Analysis

The signs that are closely related to the field of calculus and analysis are shown in Section 5.0.3 of the Mathematics Symbol Set.

The signs for vertical bar and double vertical bar belong to the category of Grouping Signs as well as to the category of Calculus and Analysis.

The root of the infinity sign is the digit 8, suggestive of the fact that it is an 8 on its side in print. It is sometimes referred to as the "lazy 8."

The integral sign frequently carries limits of integration as a subscript and a superscript. These sometimes have the appearance of underscript and overscript modifiers. However, the integral sign is too thin to support underscripts and overscripts. Therefore, despite appearances, these should be transcribed as subscripts and superscripts.

Contour integrals have various closed shapes superposed on them. When this occurs, the integral sign should be followed by the overstrike indicator ⠠⠠⠠ (5,12346) which, in turn, should be followed by the superposed shape.

The letters in the three "limit" entries are printed in the host type as is the case for all function abbreviations. An alternative for the upper limit is the letters lim with an overscript bar. Similarly, the lower limit is represented by the letters lim with an underscript bar. All three limits may have other underscripts.

log and ln are function abbreviations printed in the host type. Most of the time, log without a subscript is understood to have 10 as its base. At other times, 10, e , or some other base may be attached as a subscript to log. ln without a subscript is always understood to have e as its base. A common transcriber error is to mistake ln for ln . Note that both letters in ln are in the host type. If ln were intended, n , being a variable, would be printed in the mathematical italic type. Noting this rather subtle distinction would avoid this error.

The principal use of the prime sign in calculus is to denote the order of the derivative. Thus, $f'(x)$ denotes the first derivative of $f(x)$, $f''(x)$ denotes the second derivative of $f(x)$, etc. After the third derivative, it is customary to use Roman numbers enclosed within parentheses as superscripts to denote the order of the derivative. Thus $f^{(iv)}(x)$ denotes the fourth derivative of $f(x)$.

All the function abbreviations in trigonometry are also prominent in calculus.

Nabla is an inverted upper-case delta. Therefore, in NUBS, the roots of delta and nabla are up-down symmetric as these symbols are in print. They both have the same prefix.

The partial derivative sign is sometimes referred to as "round d."

5.4 Comparison Signs (continued)

The basic comparison signs were presented in Chapter 3, Section 3.0.3. Other comparison signs, together with their tactile graphics, are presented in Section 5.0.4 of the Mathematics Symbol Set.

In the signs for "less than or equal" and "greater than or equal," the bottom line is sometimes horizontal instead of parallel to the line above it; sometimes it is replaced by a complete equals sign. NUBS does not distinguish between these variant notational forms of these signs; the differences have no notational significance.

The stroke in the negative comparison signs is sometimes oblique and sometimes vertical. NUBS does not recognize this difference since it has no notational significance.

Comparison signs are sometimes called relational operators.

5.5 Computer and Calculator

Most modern arithmetic and algebra textbooks and computer and calculator manuals present a sequence of keystrokes by displaying a row of keytop legends. Legends other than digits are printed on a picture of a keytop. Digits are often displayed without being superposed on the picture of a keytop.

NUBS provides a method for showing a sequence of keystrokes when such a sequence is present in print. A keystroke which is the picture of a keytop with its legend is introduced in NUBS by the braille character ⠠ (246). In other notational contexts, this character is either the termination indicator or the end-cancellation indicator. When it is the first character of a keystroke, it cannot be interpreted as a termination indicator. Neither can it be interpreted as

an end-cancellation indicator which is always placed either above or below the notation to be cancelled. Therefore, using this symbol to introduce a keystroke is unambiguous.

After this begin-keytop-legend indicator, we write the keytop legend using standard NUBS notation. A space follows the keytop legend before brailleing the next keystroke. Keystrokes are notational and spatial and must be treated accordingly.

EXAMPLES (showing keystrokes)

1 Press +/- ⠠⠠⠠⠠⠠⠠

to change the sign of the number. (The legend on the keytop is +/- .)

2 2.41 EE 0 ÷ 7.4 EE 21 =

⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠

(A sequence of keystrokes in which some are numbers and some are not.)

The result of a calculation is displayed on the screen of the calculator. The symbol ⠠⠠⠠⠠⠠⠠ (12346) is used in NUBS to introduce this display. In other contexts, this symbol is the begin-cancellation indicator. However, as such, it must be placed above or below the notation to be cancelled. Therefore, the proposed symbol cannot be confused with the begin-cancellation indicator. The "rectangular" shape of this symbol suggests the rectangular shape of the screen. The information displayed is presented in standard NUBS notation. Like a sequence of keystrokes, the screen display of a calculator is both notational and spatial.

EXAMPLE (showing the screen display of a calculator)

3 3.2568 e-22 ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠

5.6 Delimiters (continued)

In Chapter 2, Section 2.3, we discussed delimiters and presented some examples. It is recommended that you review that section now. Here is further discussion and more examples.

The space is not a delimiter when it occurs between a function name or a function abbreviation and its argument. This holds even if the function name or the function abbreviation has subscripts, superscripts, overscripts, underscripts, modifiers, or any combination of these.

The space is not a delimiter when it occurs between a geometric shape and the enumeration of its points or vertices.

EXAMPLES (in which the space is not a delimiter)

- 1 $\cos x$
- 2 $\log_{10} 2.7183 \dots$
(The space after \log_{10} is not a delimiter even though log has a subscript.)
- 3 $\sin^{-1} \frac{1}{2} = \frac{p}{6}$ (None of the three spaces is a delimiter. Why?)
- 4 $\lim_{x \rightarrow 0} f(x)$ (The space is not a delimiter even though lim has an underscript.)
- 5 ΔABC and ΔXYZ are congruent.
(Only the spaces surrounding "and" are delimiters. Explain.)

5.7 Fractions and Radicals (continued)

The NUBS basics for representing fractions and radicals were presented in Chapter 3; it is recommended that you review those topics at this time before proceeding to more advanced issues.

Other codes have attempted to avoid fraction indicators by enclosing the terms of the fraction within parentheses or other enclosures. However, if the numerator of a fraction is long and the denominator is short, you will not know that you are dealing with a fraction until you encounter the fraction bar near the end of the expression. The quadratic equation is a classic case in point:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \dots \dots \dots$$

If there were a left parenthesis instead of a begin-fraction indicator in this expression, you would not know that you are dealing with a fraction until you encountered the fraction bar within two symbols from the end of the fraction. Your mental orientation to the notation at hand may be quite different when you know that you are dealing with a fraction than when you are coping with the same expression but without that information. Besides, using enclosures of any kind would be a violation of one of the NUBS principles and guidelines which mandates that no enclosures be used in braille, either real or phantom, when there are no enclosures in print.

$$x^{\frac{1}{2}} \quad \dots \dots \dots$$

shows that fractions can be in the superscript position. They can also be in the subscript position; they can be part or all of a radicand. On the other hand, fractions can contain subscripts, superscripts and radicals.

The slash, as a fraction bar or for other uses, is discussed in Chapter 3.

5.7.1 Fractions of Higher Order

It may happen that the numerator or the denominator of a fraction, or both, are themselves fractions or contain fractions. When this occurs, we are dealing with *higher-order fractions*.

A *simple fraction* is one in which neither the numerator nor the denominator is (or contains) a fraction. A simple fraction is said to be of order 0. For purposes of NUBS only, the order of a fraction is not increased if the only fractions that it contains are at the subscript level, the superscript level, or both.

A *complex fraction* is one in which the numerator, the denominator, or both are (or contain) only simple fractions. A complex fraction is said to be of order 1. The term "complex" is also used in a generic sense to apply to any fraction of order higher than 0. That is, "complex" is used as the opposite of "simple" when referring to fractions.

A *hypercomplex fraction* is one in which the numerator, the denominator, or both are (or contain) a complex fraction. A hypercomplex fraction is said to be of order 2.

In theory, it is possible to have fractions of any order whatever. As a practical matter, we frequently encounter complex fractions, we sometimes encounter hypercomplex fractions, and we almost never encounter any fractions of higher order than 2. Continued fractions (discussed in Section 5.7.3) are an exception to this statement, since they frequently are of order greater than 2. For purposes of NUBS only, we do not assign any order of complexity to a continued fraction.

Since a fraction of higher order contains more than one fraction bar, it is necessary to identify the main fraction bar. In print, this is usually the fraction bar that is the longest in the entire construct, the one that is abreast of a reference label, a comparison sign, or some other sign of operation, or the one which is printed in type heavier than the others.

NUBS provides a *complexity indicator* to inform the reader of the complexity of the fraction with which he is dealing. This is another instance in which NUBS implements the *just-in-time principle*.

RULES (regarding the complexity of a fraction)

- a *These rules apply only when representing a fraction linearly.*
- b *The complexity indicator must not be used with simple fractions.*
- c *The complexity indicator must be used once before the begin-fraction indicator, the end-fraction indicator, and the fraction bar in the case of a complex (first order) fraction.*
- d *The complexity indicator must be used twice before the begin-fraction indicator, the end-fraction indicator, and the fraction bar in the case of a hypercomplex (second order) fraction.*

- e *In general, in the case of a fraction of order n , the complexity indicator must be used n times before the begin-fraction indicator, the end-fraction indicator, and the fraction bar of such a fraction.* Fractions of order greater than 2 are almost never encountered.

EXAMPLES (regarding complex fractions)

1 $\frac{\frac{3}{8}}{5}$

2 $\frac{1/2}{2\frac{2}{3}}$

3 $\frac{2/3}{3/2}$

4 $\frac{\left(\frac{1+x}{1-x}\right)^2}{\left(\frac{1+x}{1-x}\right)^3}$

Example 1 is a complex fraction whose numerator is a simple fraction.

Example 2 contains a mixed number in its denominator. The fractional part of this mixed number is a simple fraction, causing the entire construct to be treated as a complex fraction.

Example 3, despite appearances, is a simple fraction. The slashes are merely signs of operation and are treated just like a plus sign or a minus sign might be.

In Example 4, the presence of the parentheses does not obscure the fact that there are simple fractions both in the numerator and in the denominator, so that the entire construct is a complex fraction and is so presented.

5.7.2 Spatial Representation of Fractions

NUBS provides for the representation of fractions in a form which simulates their appearance in print. Fractions presented in this way are called *spatial fractions*, and the fraction bar in such fractions is called a *spatial fraction bar*.

There are several situations in which it is appropriate to represent fractions spatially:

- When fractions are first introduced to young children and for a short while thereafter;
- When the structure of a complex fraction is more clearly displayed spatially than linearly;
- In the representation of a continued fraction;

- When performing cancellation or other computation with fractions.

Although continued fractions are spatial, they are excluded from this discussion.

RULES (regarding spatial fractions)

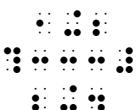
- A spatial fraction is a spatial arrangement; it must be separated from surrounding text or from other spatial arrangements by a blank line both above and below the spatial fraction.*
- A spatial fraction bar must be simulated by one or more braille characters composed of dots 25. This series of components must be long enough to cover the longer of the terms of the fraction.*
- A spatial fraction must be positioned so that its main spatial fraction bar is abreast of a label, a comparison sign, or a sign of operation, if these are present.*
- The begin-fraction and the end-fraction indicators at the ends of spatial fraction bars must be represented without complexity indicators; the spatial disposition of the fraction and the complexity indicators in its subsidiary linear fractions (if any) supply the requisite information to the braille reader.*
- It is permissible for a spatial fraction to have subsidiary fractions (of lower order) which are represented linearly.*
- When subsidiary fractions are represented linearly, they must be represented as self-contained local entities, without regard to the larger structure of which they are a part. In particular, complexity indicators must be used only as they apply to the linear portion of the notation.*
- The first character in each term of a fraction must be aligned with the first component of the spatial fraction bar with which it is associated.*
- A runover, when required, must be indented two cells beyond the expression of which it is a part.*
- If a term in a spatial fraction is a single digit, that digit must be preceded by the numeric indicator ⠠ (3456). If a term in a spatial fraction is an isolated lower-case letter from a to j (occupying a single cell), that letter must be preceded by the notational indicator ⠨ (56). In these cases, the numeric indicator and the notational indicator serve as dot locators.*

Note that the rule regarding the centering of the terms in a spatial fraction and their possible runovers has been changed from the 1972 Nemeth Code. There are also some other minor changes from the 1972 Nemeth Code.

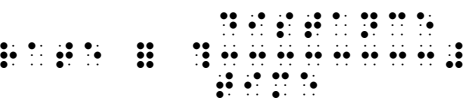
EXCEPTION

Fraction indicators must not be used in a continued fraction.

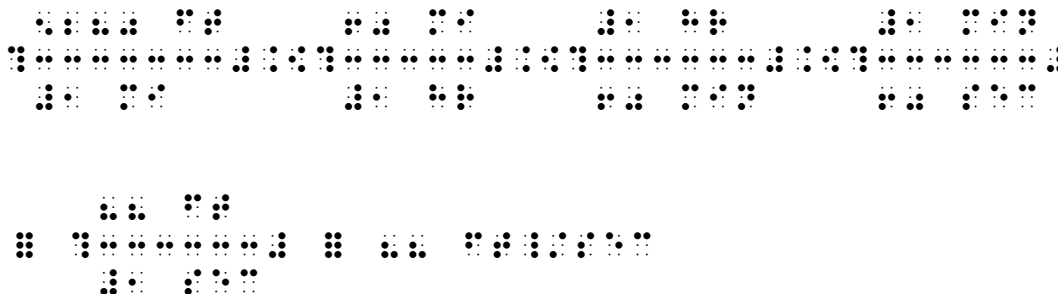
EXAMPLES (regarding spatial simple fractions)

1 $\frac{1+2}{2+4}$ 

2 $\frac{1}{c}, \frac{x}{y}$ 

3 $rate = \frac{distance}{time}$ 

4 $\frac{5280 ft}{1 mi} \times \frac{60 mi}{1 hr} \times \frac{1 hr}{60 min} \times \frac{1 min}{60 sec} = \frac{88 ft}{1 sec} = 88 ft/sec$



Example 1 is the kind that might be encountered in an elementary grade when fractions are first being introduced to young children.

Example 2 shows how isolated digits and letters are handled.

Example 3 consists of words. But since it is displayed, it is notational so that it contains no contractions.

Example 4 is a typical example of a technique called "dimensional analysis." When these fractions are displayed spatially rather than linearly, it is easy to see that each unit abbreviation in the denominator of one fraction "cancels" the same unit abbreviation in the numerator of the next fraction. The only abbreviations that remain are the "ft" in the numerator of the first fraction and the "sec" in the denominator of the last fraction. With this spatial display, it is also easy to see that 60 occurs once among the numerators and twice among the denominators. By cancelling one pair of 60's we are left with 5280 in the

numerator and 60 in the denominator. Doing the arithmetic of dividing 5280 by 60 (either mentally, or on a piece of paper, or on a calculator), we obtain 88 as the numerical result. Putting all of this together, we find that 60 miles per hour (the second fraction) is the same as 88 feet per second.

EXAMPLES (regarding spatial complex fractions)

1

$$\frac{1 - \frac{1}{4}}{1 - \frac{5}{5}}$$

2

$$\frac{(1-x) \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1-x)}{(1-x)^2} \div 1 + \left(\frac{2x}{1-x}\right)^2$$

3

4

Example 1 is the spatial representation of a hypercomplex fraction. The numerator is the quotient of two mixed numbers and so, in itself, is a complex fraction. This complex fraction is divided by 5, making the entire construct a hypercomplex fraction.

Examples 2, 3, and 4 are alternative ways of representing the same print information. In Example 2 the representation is entirely linear even though it is included in a group identified as spatial complex fractions. Example 3 is partly linear and partly spatial. Example 4 is entirely spatial.

When requested to find

$$\frac{d}{dx} \tan^{-1} \frac{2x}{1-x}$$

this notation is the first step in working out the solution to finding this derivative.

5.7.3 Continued Fractions

A *continued fraction* is one in which each denominator except possibly the last one is the sum of a whole number and a fraction. See example below.

RULES (regarding continued fractions)

- a *A continued fraction must be represented spatially.*
- b *A continued fraction is a spatial arrangement and thus requires a blank line above and below it.*
- c *A continued fraction must be positioned so that its first fraction bar is abreast of a label, a comparison sign, or other operation sign, if any.*
- d *Fraction indicators must not be used in the representation of a continued fraction.*
- e *Each fraction bar must be long enough to cover all of the notation below it which is part of the continued fraction.*

EXAMPLE (of a continued fraction)

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}}$$

The remainder of this section is aimed at those with a little flare for mathematics and is optional for the rest of us. If you have any mathematical inclination, you might wish to see how this continued fraction operates to evaluate the square root of 2. Note that every denominator is 2 plus a fraction. However, to start with, we have 1 plus a fraction. Let us change this initial 1 to a 2 to make the continued fraction uniform and note that the new continued fraction represents $1 + \sqrt{2}$ $\therefore \therefore \therefore \therefore \therefore$

We start with a denominator of 2 and denote it by x_0 , that is,

$$x_0 = 2 \quad \therefore \therefore \quad \therefore \quad \therefore$$

It is not difficult to see that the next approximation is:

$$x_1 = 2 + \frac{1}{x_0} \quad \therefore \therefore \quad \therefore \quad \therefore \therefore \therefore \therefore \therefore \therefore \therefore$$

and the one after that is:

$$x_2 = 2 + \frac{1}{x_1} \quad \therefore \therefore \quad \therefore \quad \therefore \therefore \therefore \therefore \therefore \therefore \therefore \therefore$$

In general, we have:

$$x_k = 2 + \frac{1}{x_{k-1}} \quad \therefore \therefore \therefore \quad \therefore \quad \therefore \therefore \therefore \therefore \therefore \therefore \therefore \therefore \therefore \therefore$$

It is best to work with a calculator. Since the calculator that is part of the Freedom Scientific family of notetakers is popular among blind users, we will use that calculator. If you are using a different one, you will have to make a slight adjustment to accommodate the one you are using.

We begin by entering 2 into the calculator and display it. Thereafter we enter

$$2 + \frac{1}{r} \quad \therefore \therefore \therefore \therefore \therefore \therefore \therefore$$

repeatedly, displaying the result each time, to obtain the successive approximations. On the calculator we are using, r denotes the active register (in which the results are developed.) When you press $\therefore \therefore \therefore \therefore$ (chord-e), the result of the latest calculation is displayed. We find the successive approximations to be:

$$\begin{aligned} x_0 &= 2 \\ x_1 &= 2.5 \\ x_2 &= 2.4 \\ x_3 &= 2.416666666667 \\ x_4 &= 2.413793103448 \end{aligned}$$

$$\begin{aligned}
x_5 &= 2.414285714286 \\
x_6 &= 2.414201183432 \\
x_7 &= 2.414215686275 \\
x_8 &= 2.414213197970 \\
x_9 &= 2.414213624895 \\
x_{10} &= 2.414213551646 \\
x_{11} &= 2.414213564214 \\
x_{12} &= 2.414213562057 \\
x_{13} &= 2.414213562427 \\
x_{14} &= 2.414213562364 \\
x_{15} &= 2.414213562375 \\
x_{16} &= 2.414213562373 \\
x_{17} &= 2.414213562373
\end{aligned}$$

Remember that these are successive approximations to $1+\sqrt{2}$. Note that the approximations alternate between being smaller and larger than the true value of $1+\sqrt{2}$. We stop when two successive approximations are equal; at that point we have reached the precision limit of our calculator. In this case, we have the approximation of $1+\sqrt{2}$ correct to 12 digits after the decimal point. Subtracting 1 from the value of x_{17} , we have the value of $\sqrt{2}$ correct to 12 digits after the decimal point, namely:

$$\sqrt{2} = 1.414213562373$$

This is by no means the fastest way of computing $\sqrt{2}$. It is, however, an interesting method. Furthermore, the continued fraction for $\sqrt{2}$ was given by revelation; no attempt was made to show how we arrived at this continued fraction. If you had to compute $\sqrt{3}$ by using a continued fraction, you would first have to find the continued fraction that represents $\sqrt{3}$. This book is not the place to teach you how to do that.

5.7.4 Indexed Radicals

A radical is said to be *indexed* if it carries a superscript. If the superscript is 3, we are dealing with the cube root; if the superscript is n , we are dealing with the n^{th} root, etc. When a radical sign carries no index, the index is implicitly understood to be 2, so in that case, we are dealing with the square root.

In NUBS, the index of a radical is attached to the radical sign in the form of a superscript. Note that this differs from the method of indexing a radical as provided in the 1972 Nemeth Code.

RULE (regarding indexed radicals)

If a radical is indexed, the index must be attached to the radical sign in the form of a superscript.

EXAMPLES (regarding indexed radicals)

1 $\sqrt[3]{2}$ ⋮⋮⋮⋮⋮⋮ (the cube root of 2)

2 $3\sqrt[3]{x+y}$ ⋮⋮⋮⋮⋮⋮⋮⋮⋮ (3 times the cube root of x plus y)

3 $\sqrt[n]{a}$ ⋮⋮⋮⋮⋮⋮ (the n^{th} root of a)

4 $\sqrt[m+n]{p+q}$ ⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮ (the m plus n^{th} root of p plus q)

5.7.5 Radicals of Higher Order

Just as in the case of fractions, it may happen that a radicand is itself a radical expression or contains one or more radical expressions. In such situations, we are dealing with *nested radicals*. A *simple radical* is one in which there are no inner radicals. A simple radical is said to be of order 0. For purposes of NUBS only, the order of a radical is not increased if the only radicals that it contains are at the subscript level, the superscript level, or both. An indexed radical can be a simple radical if it contains no inner radicals.

A radical expression that contains inner radicals only one level deep is said to be of order 1. A radical expression that contains inner radicals that are at most two levels deep is said to be of order 2. A radical expression that contains inner radicals that are at most n levels deep is said to be of order n . In theory, it is possible to have radicals of any order whatever. As a practical matter, we seldom encounter radicals of order higher than 2.

In a manner parallel to the treatment of higher-order fractions, we precede the radical sign and its radical terminator with the complexity indicator, repeated as many times as the order of the radical. Thus, a radical of order 0 requires no complexity indicators; a radical of order 1 requires a single complexity indicator before the radical sign and its terminator; a radical of order 2 requires two complexity indicators before the radical sign and its terminator, etc.

Note that this method of handling radicals of higher order differs from the method provided for this purpose in the 1972 Nemeth Code. This revised method results in the treatment of higher-order fractions and higher-order radicals in a more uniform manner. This revised method also implements the *just-in-time principle*, giving the reader information about the order of the radical with which he must deal just as that radical expression begins.

RULES (regarding the complexity of a radical expression)


- a *The complexity indicator must not be used with simple radicals.*
- b *The complexity indicator must be used once before the radical sign and its terminator in the case of a first-order radical.*

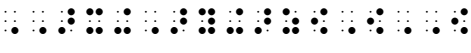
- c *The complexity indicator must be used twice before the radical sign and its terminator in the case of a second-order radical.*
- d *In general, in the case of a radical of order n , the complexity indicator must be used n times before the radical sign and its terminator.*

EXAMPLES (regarding higher-order radicals)

1 $\sqrt{x + \sqrt{x + y + y}}$ 

2 $\sqrt[3]{x^2 + \sqrt[3]{x^2 + y^2} + y^2}$


3 $\sqrt{\sqrt[3]{x}} = \sqrt[3]{\sqrt{x}}$ 

4 $\sqrt{x + \sqrt{y + \sqrt{z}}}$ 

5.7.6 Spatial Representation of Radicals

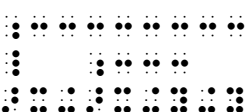
It is possible to represent radicals spatially. A spatial representation is appropriate when the complexity of the radical expression is such that a spatial representation is clearer than a linear representation.

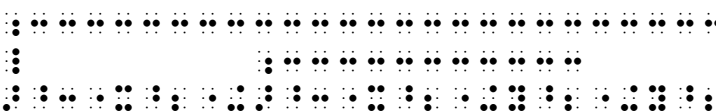
RULES (regarding the spatial representation of radicals)

- a *A radical expression displayed spatially is a spatial arrangement and requires a blank line above and below it.*
- b *In a spatial representation of a higher-order radical expression, that expression must be positioned so that the highest-order radical sign is abreast of a label, a comparison sign, or sign of operation.*
- c *Other than the radical sign and the radicand, there are two structures that endow the expression with its spatiality, namely its ascenders and its vincula.*
- d *Each ascender is simulated by a vertical column of dots in which each component but the topmost one consists of dots 456. The topmost component consists of dots 56. All the components of an ascender must occupy the same column of cells as the radical sign with which the ascender is associated. The topmost component of an ascender must be one braille line higher than the radicand with which it is associated.*

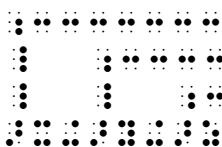
- e Each vinculum is simulated by a horizontal row of dots in which each component consists of dots 25. The vinculum must occupy the same braille line as the topmost component of its ascender. Its first component must occupy the cell after the topmost component of its ascender, and the vinculum must be long enough to cover the radicand with which it is associated.
- f The order of the radical is not indicated in a spatial representation, since the extent and position of each ascender and vinculum provides this information.

EXAMPLES (showing the spatial representations of the four examples of Section 5.7.5)

1a $\sqrt{x + \sqrt{x + y} + y}$ 

2a $\sqrt[3]{x^2 + \sqrt[3]{x^2 + y^2} + y^2}$ 

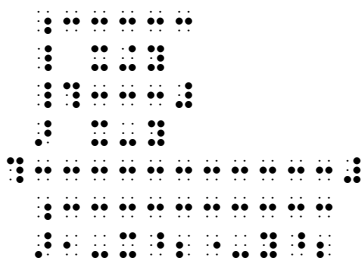
3a $\sqrt{\sqrt[3]{x}} = \sqrt[3]{\sqrt{x}}$ 

4a $\sqrt{x + \sqrt{y + \sqrt{z}}}$ 

5.7.7 Fractions and Radicals Together in a Spatial Representation

It is possible to have spatial arrangements in which radicals are contained within fractions or in which fractions are contained within radicals. The method is straightforward; nothing more is required than to apply the rules for the representation of spatial fractions and spatial radicals. We present one example as a model.

EXAMPLE (showing the spatial representation of radicals within a fraction)

$$\frac{\sqrt{\frac{x+y}{x-y}}}{\sqrt{1-x^2-y^2}}$$


5.8 Functions and Their Abbreviations

A list of function names and their abbreviations is presented in Section 5.0.7 of the Mathematics Symbol Set. The principal trigonometric function abbreviations are listed separately in Section 5.0.16. These function abbreviations have been accumulated over a period of time from several branches of mathematics. No doubt, others will emerge in due time with more experience. If discovered, they should be included in the list and subjected to the same rules that already apply to the existing list.

Function abbreviations are printed in the host type. Sometimes, for a special reason, the first letter is printed as an upper-case letter. In such a case, the indicator for an upper-case single letter (dot 6) should be used.

Function abbreviations are notational and contractions should not be used in their representation. Function names, when used in an expository passage rather than in a mathematical expression, may be contracted.

When a subscript, whether numeric or otherwise, is attached to the last letter of a function abbreviation, it should be regarded as applying to the whole function abbreviation, not just to its last letter.

EXAMPLES (regarding function abbreviations)

1 sin x ⠠⠠⠠⠠⠠⠠⠠

2 cos² x ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

3 e^{sin x} ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

4 arc AOB ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

5 log₁₀ x ⠠⠠⠠⠠⠠⠠⠠⠠⠠

6 $\lim_{x \rightarrow 0} f(x)$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

5.8.1 Spacing with Function Abbreviations

RULES (for spacing with function abbreviations)

- a *A space must be left between a function abbreviation and its argument.*
- b *If the function abbreviation carries a subscript, a superscript, an underscript, an overscript, a direct modifier, or any combination of these, there still must be a space before its argument.*
- c *There must also be a space after its argument, taking into account punctuation, if any.*
- d *If two or more function abbreviations are consecutive, follow print to determine whether they should be separated by a space.*

EXAMPLES (regarding spacing with function abbreviations)

- 1 $\sin x + y$ ⠠⠨⠠⠽ ⠠⠒ ⠠⠒⠠⠽
- 2 $\sin p/3$ ⠠⠨⠠⠽ ⠠⠒⠠⠒⠠⠽
- 3 $\sin 30^\circ \cos 45^\circ$ ⠠⠨⠠⠽ ⠠⠒⠠⠒⠠⠽ ⠠⠒⠠⠒⠠⠽ ⠠⠒⠠⠒⠠⠽
- 4 $\sin x \cos y$ ⠠⠨⠠⠽ ⠠⠒ ⠠⠒⠠⠽ ⠠⠒
- 5 $2\sin x + 3\cos y$ ⠠⠒⠠⠨⠠⠽ ⠠⠒⠠⠒⠠⠽ ⠠⠒ ⠠⠒⠠⠒⠠⠽ ⠠⠒
- 6 $\arcsin x$ ⠠⠒⠠⠒⠠⠽ ⠠⠨⠠⠽ ⠠⠒ (In print, there is a space in the function name.)
- 7 $\log \sin 60^\circ$ ⠠⠒⠠⠒⠠⠽ ⠠⠒⠠⠒⠠⠽ ⠠⠒⠠⠒⠠⠽ (no space in the function name in print)

5.9 Geometry

5.9.1 Geometric Modifiers

Section 5.0.8.1 of the Mathematics Symbol Set lists the modifiers commonly found in geometry.

These modifiers are used in the notation for lines, half-lines, and line segments. These geometric objects are usually identified by a pair of upper-case letters with the appropriate modifier placed above them.

In braille, the mechanism for showing modification must be used, and the letters which identify the line or line segment must be individually capitalized.

In the case of half-lines and line segments as shown in print, a solid dot is used at the end of the modifier if the endpoint is included, and a hollow dot is used if the endpoint is excluded.

EXAMPLES (of geometric modifiers)

- 1 ⠠⠨⠨⠨⠨⠨⠨⠨⠨ (full-line AB)
- 2 ⠠⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨ (The half-line PQ excludes its endpoint.)
- 3 ⠠⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨ (The line segment CD in which C is excluded and D is included.)

5.9.2 Geometric Shapes




Many of the shapes that are the subject matter of plane geometry are represented by little icons rather than by words which identify those shapes. A list of the most common icons together with their tactile graphics is presented in Section 5.0.8.2 of the Mathematics Symbol Set. They all have the common compound prefix ⠠⠨⠨ (456,5).

Note how regular polygons are represented. The root specifies the number of sides (which are all of equal length). This technique applies only to regular polygons; it may be applied to other regular polygons not listed. Thus, a triangle that is not equilateral is represented as ⠠⠨⠨⠨ (456,5,2345), whereas an equilateral triangle is represented as ⠠⠨⠨⠨ (456,5,25).

A shape may be shown as hollow, shaded, or filled-in (solid). If it is shown as hollow, nothing more than the shape shown in Section 5.0.8.2 of the Mathematics Symbol Set is required. If the shape is shown as shaded, the root of the shape symbol is preceded by ⠠⠨ (46). If the shape is shown as solid, the root of the shape symbol is preceded by ⠠⠨ (456).

In addition, a shape may be shown with a boundary or without a boundary. Of course, a shape that is hollow always has a boundary. If a shaded or a solid shape has a boundary, only the mechanism described in the above paragraph is necessary. But if a shaded or a solid shape has no boundary, the additional prefix ⠠⠨ (4) is placed before the shape indicator ⠠⠨⠨ (456,5).

EXAMPLES (showing geometric shapes)

- 1 ⠠⠨⠨⠨⠨ (a solid rectangle) 
- 2 ⠠⠨⠨⠨⠨ (a shaded square) 
- 3 ⠠⠨⠨⠨⠨ (a shaded circle without a boundary) 

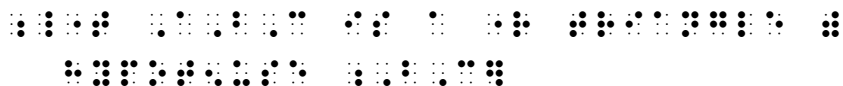
When referring to points or vertices of a geometric figure in which upper-case letters are used to name those points or vertices, those letters must be individually capitalized.

EXAMPLES (showing how points or vertices of a geometric figure are represented)

- 4 Lines AB and CD intersect at P.

$\text{⠠⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨⠨}$

5 ΔABC is a right triangle with hypotenuse BC.



5.10 Grouping Signs (continued)

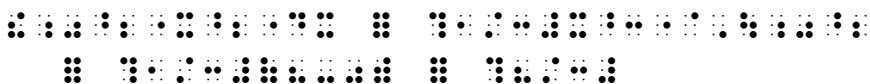
The basic grouping signs together with their tactile graphics are presented in Section 3.0.9 of the Basic Literary Symbol Set. To those we now add the more technical grouping signs as listed in Section 5.0.9 of the Mathematics Symbol Set.

Most of the time, grouping signs occur in pairs. However, there are times when only the left component of a pair of grouping signs is present without its matching right component, and vice versa. When this occurs, the braille must follow the print.

Vertical bars are mostly used to convey the meaning of "absolute value." In that role, its two identical components enclose the intended expression. However, a vertical bar by itself may be used at the right of an expression and carry a subscript and a superscript to indicate the limits for evaluation. The right bracket by itself is also used for this purpose.

EXAMPLE (showing the use of a vertical bar other than for absolute value)

$$1 \quad \int_0^2 x^2 dx = \frac{1}{3} x^3 \bigg|_0^2 = \frac{1}{3} (8-0) = \frac{8}{3}$$



A pair of brackets which enclose only a space is sometimes used to represent the empty set.

The double vertical bar is used with the meaning of "norm."

Here are some examples, all involving grouping signs. Justify the way in which each example is transcribed.

EXAMPLES (showing grouping signs)

2 (i.e.) $\{ \}$

3 (s.a.s. = s.a.s.) $\{ \}$

4 {Wed., Thurs., Fri.}

5 {Mary, Sally, Jean}

Upper half-brackets are used in mathematics for the "ceiling" function, and lower half-brackets are used for the "floor" function. These half-brackets are also used for other miscellaneous purposes. In the mathematics of finance, the upper right half-bracket is used as a subscript with an expression inside the bracket and an expression outside and to the right of the bracket.

EXAMPLE (from the mathematics of finance)

19 $A_{n \lceil i}$

Extended parentheses are used to enclose a binomial coefficient. In print, the components of a binomial coefficient are written one above the other. In NUBS, a binomial coefficient is represented by an extended set of parentheses in which the upper component is followed by the lower component, separated by a space.

EXAMPLE (of binomial coefficient)

20 $\binom{n!}{(n-k)!}$

In print, extended grouping signs are used to span a determinant, a matrix, or a system of equations. In NUBS, the extended grouping signs should be placed on every line to the left, to the right, or on both sides of the notation which it spans, following print. Extended grouping signs should not be used to span a vertically-stacked fraction or to span a tall integral sign which spans a vertically-stacked fraction, or a stacked binomial coefficient. If the system with its extended grouping signs is associated with an identifier, a comparison sign, or an operation sign, it should be so positioned that its top line is abreast of these.

RULE (for the placement of a system spanned by extended grouping signs)

A system spanned by extended grouping signs should be placed so that its top line is abreast of a label, a comparison sign, or an operation sign, if any. If the system is punctuated, the punctuation mark should be placed alongside the top line.

EXAMPLES (of extended grouping signs)

21 $\begin{cases} x + y = 2 \\ x - y = 0 \end{cases}$

(a system of two equations spanned on the left by an extended brace)

22 $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

(The evaluation of a 2 x 2 determinant is enclosed by vertical bars. The right side of the equation in print is centered on the extended vertical bar; in NUBS it is abreast of the top line of the extended vertical bar.)

$$23 \quad y = \begin{cases} x, & \text{if } x \leq 0 \\ 0, & \text{if } x > 0 \end{cases}$$

(a piecewise-defined equation whose components are spanned on the left by an extended bracket)

$$24 \quad x = \begin{bmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

(A 3 x 3 matrix is spanned on both sides by extended brackets. Isolated digits require the numeric indicator. In print, the left side of the equation is centered on the extended left bracket. The period on the right is centered on the extended right bracket. In NUBS, both of these are abrest of the top line of the matrix.)

Sometimes a system of equations or a piecewise-defined equation has a comment, narrative or notational, centered so as to apply to the system, but without any extended grouping sign at all. When this happens, NUBS requires an extended transcriber enclosure to span the system so as to associate the comment with the system. If the comment is to the left of the system, the left extended transcriber enclosure is required; if at the right, the right extended transcriber enclosure is required.

EXAMPLE (showing the use of an extended transcriber enclosure)

$$25 \quad a = \frac{x+y}{x-y}$$

$$\begin{aligned} -1 < x < 1, \\ -1 < y < 1 \end{aligned}$$

$$b = \frac{x-y}{x+y}$$

(In print, the conditions for x and y are on two lines centered between two equations with no extended grouping sign.)

5.11 Logic and Set Theory

Logic and set theory are closely related branches of mathematics and thus share many symbols. Most of those symbols can be classified either as operation signs or as comparison signs, and have been included in their respective categories in the Mathematics Symbol Set together with their tactile graphics. They are governed by the same rules that we have already encountered regarding other operation signs and comparison signs.

Note that the symbols for "meet" and "join" are top-to-bottom symmetric in print; they are likewise top-to-bottom symmetric in NUBS. The same observation holds for the signs for "union" and "intersection."

Note that the prefix \neg (46) denotes negation. It is placed either before the first character of the principal sign to achieve negation, or it replaces that first character.

The comparison signs in the Logic and Set Theory list have been duplicated in the Signs of Comparison list, and the operation signs in the Logic and Set Theory list have been duplicated in the Operation Signs list. Four entries in the Logic and Set Theory list are neither comparison signs nor operation signs.

5.12 Special Letters (continued)

The special letters recognized by NUBS have been assigned to seven categories. These are all listed in the Mathematics Symbol Set together with their graphic representation.

5.12.1 Crossed

The crossed b is used in computer displays to denote a blank when it is important to count the actual number of blanks.

The crossed h is used to represent Planck's Constant in the field of quantum mechanics.

5.12.2 Global Operation Signs

The large upper-case boldface sigma is used to represent the sum of a large number of terms. Similarly, the large upper-case boldface pi is used to represent the product of a large number of factors. Other global operation signs that are not letters are entries in the Logic and Set Theory list.

5.12.3 Hebrew

Two Hebrew letters, aleph and beth, are used in the branch of mathematics that deals with transfinite numbers. The 1972 Nemeth Code lists letters of the entire Hebrew alphabet, both in their printed and script forms. However, experience has proved this to be unnecessary.

5.12.4 Miscellaneous

The two letters, partial derivative and Weierstrass p , are used in the branch of mathematics that deals with calculus and analysis.

5.12.5 Number Sets

Mathematics employs five letters, each in the same special font, to denote the principal types of numbers: the natural numbers, the integers, the rational numbers, the real numbers, and the complex numbers.

5.12.6 Obsolete Greek

Obsolete Greek letters are used in discussions concerning the ancient Greek numeration system.

5.12.7 Variant Greek

Sometimes two forms of the same Greek letter are used in an expository passage and in the accompanying notational text. When this is the case, the variant form must be distinguished from the standard form, so that NUBS provides the means for doing so.

At other times, a variant form of a Greek letter is used without mention of the alternative standard form. When this is the case, NUBS permits the variant form to be replaced by the standard form of that letter without fear of ambiguity.

5.13 Subscripts and Superscripts (continued), and Modifiers

It is characteristic of mathematical notation as well as of notation elsewhere to place some symbols at an elevated or a depressed position relative to the principal line of notation. Symbols that are elevated are called *superscripts*; symbols that are depressed are called *subscripts*. The principal line of notation is called the *base line*. Notation, either at an elevated level or at a depressed level, is called an *index* (plural, *indices*).

Subscripts and superscripts have been mentioned in an informal way in Chapter 3 as well as in several earlier sections of this chapter. It is time to give them a coherent analysis.

A superscript is introduced by the indicator $\overset{\cdot}{\cdot}$ (45). It remains in effect until it is cancelled by another level indicator or by a space. Similarly, a subscript is introduced by the indicator $\underset{\cdot}{\cdot}$ (56). It remains in effect until it is cancelled by another level indicator or by a space. The indicator $\overset{\cdot}{\cdot}$ (5) is used to return to the base line. Notice how suggestive these indicators are of their functions.

EXAMPLES (of simple subscripts and superscripts)

1 x^2 $\overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot} \overset{\cdot}{\cdot}$ (x square)

subscripts or superscripts. While there is no theoretical limit to the number of subscript or superscript levels, the practical limit is two, or at most three levels.

In NUBS, a higher-level subscript or superscript is indicated by a sequence of level indicators which leads the reader on a path beginning at the base level and ending at the higher-level symbol. Thus, ⠠⠠⠠ (45,45) takes the reader from the base level to the "up-up" level; ⠠⠠⠠ (56,45) takes him from the base level to the "down-up" level; ⠠⠠⠠⠠ (45,56,45) takes him from the base level to the "up-down-up" level, etc.

Once a level has been indicated, it remains in effect until it is terminated by another level indicator or by a space.

EXAMPLES (of higher-order indices)

1 $\int e^{-x^2} dx$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

2 e^{x^2+1} ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

3 $e^{x^2}_{+1}$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

In Example 2, the 1 is at the "up" level, abreast of the x. In Example 3, the 1 is at the base level, abreast of the e.

5.13.6 Overscripts and Underscripts

In addition to subscripts and superscripts, mathematical notation permits stacked notation. In notation of this kind, the principal expression is modified by subsidiary expressions which are written below the principal expression, above the principal expression, or both. Subsidiary expressions that are written below the principal expression are called *underscripts*; subsidiary expressions written above the principal expression are called *overscripts*. The principal expression may contain more than one underscript, more than one overscript, or more than one of both. These are written at successively more remote levels relative to the principal expression.

In NUBS, a first-level underscript is indicated by:

⠠⠠⠠ (56,12346)

A first-level overscript is indicated by:

⠠⠠⠠ (45,12346)

Successive underscript levels are indicated by using ⠠⠠ (56) as many times as required for second-, third-, etc. level underscripts and by using ⠠⠠ (45) as many times as required for second-, third-, etc. level overscripts. Whether there is one or there are several such prefixes, each sequence must be followed by the ⠠⠠ (12346) character. All the underscripts must be attended to first, before attending to the overscripts, if any. Finally, the termination indicator

If the modifier is an overscript, the expression which is modified is preceded by $\overset{\cdot}{:}$ (126); if the modifier is an underscript, the expression which is modified is preceded by $\underset{\cdot}{:}$ (146). (These are the "directly-over" and the "directly-under" indicators of the 1972 Nemeth Code.) The expression to be modified is then followed by the specific modifier. As is the case with normal underscripts and overscripts, a modified expression is a stack. In the case of modifiers, however, a termination indicator is not required; the stack ends with the last modifier. The following is a list of modifiers recognized by NUBS. If a symbol is not on this list, it must be treated in the manner of an underscript or an overscript as described in the preceding section.

Modifiers

acute accent	$\overset{\cdot}{:}$	(4,35)
asterisk	$\overset{*}{:}$	(456,246)
breve	$\overset{\text{˘}}{:}$	(456,5,1)
circumflex accent	$\overset{\text{ˆ}}{:}$	(6,26)
diaeresis	$\overset{\text{¨}}{:}$	(46,25)
dot	$\overset{\cdot}{:}$	(12456)
grave accent	$\overset{\text{˘}}{:}$	(4,26)
hat	$\overset{\text{^}}{:}$	(6,26)
left arrow	$\overset{\leftarrow}{:}$	(4,246)
macron	$\overset{\text{¯}}{:}$	(156)
right arrow	$\overset{\rightarrow}{:}$	(4,135)
ring	$\overset{\text{◌}}{:}$	(46,346)
tilde	$\overset{\text{˜}}{:}$	(46,35)
two-way arrow	$\overset{\leftrightarrow}{:}$	(4,123456)
umlaut	$\overset{\text{¨}}{:}$	(46,25)

EXAMPLES (of modifiers)

- 1 \overline{x} $\overset{\text{¯}}{:} x$ (bar over x)

- 2 \dot{x} $\overset{\cdot}{:} x$ (dot over x)

- 3 $\overline{3.142857}$ $\overset{\text{¯}}{:} 3.142857$ (a repeating decimal)

4. $\overline{\overline{x + y}}$ $\overset{\text{¯}}{:} \overset{\text{¯}}{:} x + y$
 (the conjugate of, the conjugate of x plus the conjugate of y .)

- 5 \hat{xy} $\overset{\text{^}}{:} xy$ (hat over xy)

- 6 $\underset{\rightarrow}{x}$ $\underset{\rightarrow}{:} x$ (right arrow under x .)

5.14 Trigonometry

Trigonometry is characterized by the generous use of function abbreviations. It is customary for these function abbreviations to be printed in the host type. The function abbreviation for cotangent has two commonly used forms, namely, \cot and ctn . There are other function abbreviations which are not specific to trigonometry, and these have been listed separately in Section 5.0.7 of the Mathematics Symbol Set. Other than these function abbreviations, the notation in trigonometry is the same as that used in other branches of mathematics.

Some now obsolete trigonometric function abbreviations are included in the list of Function Abbreviations.

This index cover the entire NUBS document, Chapters 0 through 8.

Each entry in the index includes the section number and the print page number. In the braille version, this is followed by an additional notation to provide the braille volume and braille page number. For example a reference of: ANGLE BRACKETS 3.2.3 49 indicates that angle brackets are addressed in Section 3.2.3 on print page 49. Since the indexed item may not occur at the beginning of the referenced section, the print page listed may not include the start of that section.

A

ABACUS AND CALCULATOR

discussion regarding 4.7 111

ACCENT MARKS

discussion regarding 3.4.1 61

in foreign languages 3.4.3 64

list of 3.0.1 37

placement of 3.4.1 61

ALPHABET

braille 0.7 6

English 3.1 46

Greek 3.9.1 85

other 3.9.2 86

ANGLE BRACKETS 3.2.3 50

ANGLICIZED WORDS 3.4 61

ANTOINE NUMBERS (See DOT-6 NUMBERS.)

APOSTROPHE 3.2.5 51

ARROWS

discussion regarding 3.7.1 67-68 and 5.1 123

list of 3.0.2 37 and 5.0.1 115

ASCII CHARACTER SET

discussion regarding 5.2.1 123

list of 5.0.2.1 115

ASTERISK 3.2.6 52

AUXILIARY NOTATION

discussion regarding 5.2.1 125

list of 5.0.2.1 116

AVOIDING PAPER CREEP

discussion regarding 4.3.2 103

example of 4.3.2 104

B

BACKSLASH 3.2.6 52

BASIC LITERARY SYMBOL SET

discussion regarding 3.0 37

list of 3.0.1-3.0.19 37-45

BINOMIAL COEFFICIENT

discussion regarding 5.10 145

example of 5.10 145

BOLDFACE INDICATORS

- for beginning a phrase 3.6 65
- for ending a phrase 3.6 65
- for one character 3.6 65
- for one word 3.6 65

BRACES 3.2.3 49**BRACKETS 3.2.3 49****BRAILLE**

- 8-dot 0.5 5
- alphabet 0.7 6
- blemishes and defects in 1.3 15
- cell 0.4 4
- characters 0.6 6
- contributions toward the advancement of civilization 1.3 14
- definition of 0.3 3
- dimensions of 0.5 5
- discussion regarding 0.3 4
- dot numbers 0.4 4
- how it is implemented 1.3 14
- jumbo 0.5 5
- lower-case *b* in 0.3 4
- microbraille 0.5 5

BRAILLE CHARACTERS

- 7 groups of 0.7 7
- assignment of meaning to 0.8 9
- rank of 0.7 9

BRAILLE SIGNS

- large 0.7 7
- lower 0.7 8

C**CALCULUS AND ANALYSIS**

- discussion regarding 5.3 125

CANCELLATION

- examples of 4.6 110
- in general 4.6 109
- indicators 4.6 110
- rules regarding 4.6 110
- when subtracting 4.5 105

CHAPTER REFERENCE LISTS

- Chapter 0 0.0 2
- Chapter 1 1.0 12
- Chapter 2 2.0 24
- Chapter 4 4.0 100

CHECK MARK 3.7.9 77**CODE vs. SYSTEM 1.5 17****COLON 3.2.8 53**

- COMMA 3.2.10 55
- COMMA AND DECIMAL POINT IN NUMBERS 3.3.5 60
- COMPARISON SIGNS
 discussion regarding 3.7.2 68 and 5.4 126
 list of 3.0.3 39 and 5..0.4 116
- COMPOUND WORDS
 definition of 3.8.3.3 84
 discussion regarding 3.8.3.3 84
 hyphenation of 3.8.3.3 84
 list of 3.8.3.3 84
- COMPUTATIONAL SCHEMES
 format requirements 4.2 100
 recognizing 4.1 100
 rules lines in 4.2 101
- COMPUTER AND CALCULATOR
 discussion regarding 5.5 126
 examples regarding keystrokes 5.5 127
 examples regarding screen display 5.5 127
- CONTINUED FRACTIONS
 discussion regarding 5.7.3 134
- CONTRACTIONS 3.8.1 80
- CURRENCY SIGNS
 discussion regarding 3.5 65
 list of 3.0.4 38

D

- DASH
 discussion regarding 3.2.9 54
 as unconditional delimiter 2.3 29
- DEFINITION OF
 braille, 0.3 6
 compound word 3.8.3.3 84
 diacritics 3.4.2 62
 hybrid construct 2.3 29
 hybrid word 2.4 31
 narrative phrase 2.5 33
 narrative word 2.4 30
 notational phrase 2.5 33
 notational word 2.4 30
 quasi-vertical fraction 3.7.8.1 74
 simple fractions 3.7.8 72
 simple radicals 3.7.9 77
 simple subscripts and superscripts 3.7.10 78
 word 2.4 30
- DELIMITERS
 conditional 2.3 27
 discussion regarding 2.3 27 and 5.6 127

- examples of 2.3 28-29 and 5.6 127
- list of 2.3 27
- when hyphen and slash are not 2.3 28
- when space is not 2.3 27

DIACRITICS

- additional letters within 3.4.2 63
- definition of 3.4.2 62
- discussion regarding 3.4.2 62
- ligatures within 3.4.2 62
- list of 3.0.1 37

DISCUSSION REGARDING

- abacus and calculator 4.7 111
- accent marks 3.4.1 61
- angle brackets 3.2.3 50
- Antoine numbers (See dot-6 numbers.)
- apostrophe 3.2.5 51
- arrows 3.7.1 67-68 and 5.1 123
- ASCII character set 5.2 123
- asterisk 3.2.6 52
- auxiliary notation 5.2.1 124
- avoiding paper creep 4.3.2 103
- backslash 3.2.6 52
- Basic Literary Symbol Set 3.0 37
- binomial coefficient 5.10 145
- braces 3.2.3 49
- brackets 3.2.3 49
- braille 0.3 4
- braille characters 0.6 6
- calculus and analysis 5.3 125
- check mark 3.7.9 77
- code vs. system 1.5 17
- colon 3.2.8 53
- column separation 4.3.1 102
- comma 3.2.10 55
- comma and decimal point in numbers 3.3.5 60
- comparison signs 3.7.2 68 and 5.4 126
- compound words 3.8.3.3 84
- computer and calculator 5.5 126
- continued fractions 5.7.3 134
- contractions 3.8.1 80-81
- currency signs 3.5 64
- dash 3.2.9 54
- delimiters 2.3 27 and 5.6 127
- diacritics 3.4.2 62
- ditto marks 3.2.4 50,51
- dot-6 numbers 3.3.2 58
- double quotes 3.2.4 50

dropped numbers 3.3.3 58
 duality problem 2.1 26
 ellipsis 3.2.7 52
 English lower-case letters 3.1.1 46
 English upper-case letters 3.1.2 46
 English upper-case phrases and passages 3.1.4 47
 English upper-case words 3.1.3 47
 exclamation mark 3.2.8 53
 feet 3.2.5 51
 fonts 3.6 65
 foreign languages 3.4.3 64
 fractions and radicals 5.7 129
 fractions of higher order 5.7.1 129
 French numbers (See dot-6 numbers.)
 geometric modifiers 5.9.1 142
 Greek alphabet 3.9.1 85
 grouping signs 3.7.3 68 and 5.10 143
 hybrid constructs 2.3 29
 hybrid word 2.4 31
 hyphen 3.2.9 53
 hyphenation 3.8.3 82
 inches 3.2.4 50,51
 keystrokes 5.5 126-127
 legal signs 3.7.4 69
 Mathematical Symbol Set 5.0 115
 metaphor 1.4 15
 minutes 3.2.5 51
 miscellaneous signs 3.7.5 70
 mixed numbers 3.7.8.4 76
 modifiers 5.13.7 153
 numbers 3.3 56
 numbers that are not subscripts 3.7.10.2 80
 numeric indicator 3.3.4 59
 numeric subscripts 3.7.10.2 80
 one-half and one-fourth 3.7.8.5 76
 operation signs 3.7.6 71
 other alphabets 3.9.2 86
 parentheses 3.2.2 48
 period 3.2.10 54
 phrases and passages 2.5 33
 prefixes 3.8.3.1 83
 prime 3.2.5 51
 prime together with indices 5.13.1 149
 principles and guidelines 1.7 18
 punctuation 3.2 48
 quasi-vertical fractions 3.7.8.1 74
 reference signs 3.7.7 72

Roman numerals 3.3.7 60-61
 seconds 3.2.4 51
 sequencing 3.8.2 81
 simple fractions 3.7.8 72
 simple radicals 3.7.9 77
 simple subscripts and superscripts 3.7.10 78
 single quotes 3.2.5 51
 slash 3.2.6 52
 slash in connection with fractions 3.7.8.3 75
 space as a non-delimiter 2.3 27
 spatial representation of fractions 5.7.2 130
 special letters 3.9.3 86
 typographic conventions 3.6.1 66
 underscore 3.2.9 54
 uniform 1.2 14
 upper numbers 3.3.1 56
 word 2.4 30
 DITTO MARKS 3.2.4 50, 51
 DOT-6 NUMBERS 3.3.2 58
 DOUBLE QUOTES 3.2.4 50
 DROPPED NUMBERS 3.3.3 58
 DUALITY PROBLEM 2.1 26

E

ELLIPSIS 3.2.7 52
 ENGLISH ALPHABET
 discussion regarding lower-case letters 3.1.1 46
 discussion regarding upper-case letters 3.1.2 46
 list of lower-case letters 3.0.5 38
 list of upper-case letters 3.0.6 38
 upper-case phrases and passages 3.1.4 47
 upper-case words 3.1.3 47
 EXAMPLES OF
 avoiding paper creep 4.3.2 104
 binomial coefficient 5.10 145
 C++ program 5.2 124
 cancellation when subtracting 4.5 106-109
 carried numbers 4.4 105
 column separation 4.3.1 103
 complex fractions (See fractions of higher order.)
 complex radicals (See radicals of higher order.)
 continued fractions 5.7.3 134
 delimiters 2.3 28-29 and 5.6 127
 font is important 2.4.1 32
 format requirements for a computational scheme 4.2 102
 fractions and radicals together 5.7.7 140
 fractions of higher order 5.7.1 130

function abbreviations 5.8 140 and 5.8.1 141
 geometric modifiers 5.9.1 142
 geometric shapes 5.9.2 142
 grouping signs 5.10 143-146
 higher-order indices 5.13.5 152
 how zeros are handled in subtraction 4.5.1 109
 hybrid constructs 2.3 29
 hybrid words 2.4 31
 hyphen and slash are not delimiters 2.3 28
 indexed radicals 5.7.4 137
 keystrokes 5.5 127
 left indices 5.13.3 150
 mixed numbers 3.7.8.4 76
 mnemonics for NUBS symbols 1.7k 20
 modifiers 5.13.7 154
 narrative words 2.4 30
 non-simultaneous indices 5.13.2 150
 notational phrases 2.5 33
 notational words 2.4 31
 numbers that are not subscripts 3.7.10.2 80
 numeric subscripts 3.7.10.2 80 and 5.13.4 151
 overstrike mechanism 5.13.6 153
 preservation of structure 1.7p 21
 primes 5.13.1 149
 quasi-vertical fractions 3.7.8.1 74
 radicals of higher order 5.7.5 138 and 5.7.6 139
 screen display 5.5 127
 simple fractions 3.7.8 73
 simple radicals 3.7.9 78
 simple subscripts and superscripts 3.7.10 79 and 5.13 148
 simultaneous indices 5.13.2 149
 slash, other uses of 3.7.8.3 75
 space is not a delimiter 2.3 27 and 5.6 127
 spatial complex fractions 5.7.2 133
 spatial complex radicals 5.7.6 139
 spatial simple fractions 5.7.2 132
 underscripts and overscripts 5.13.6 153
 EXCLAMATION MARK 3.2.8 53

F

FEET 3.2.5 51

FONTS

boldface 3.6 65
 discussion regarding 3.6 65
 examples of importance of 2.4.1 32
 italics 3.6 66
 underlined 3.6 66

upper case 3.6 66

FOREIGN LANGUAGES

discussion regarding 3.4.3 64

list of supported languages 3.4.3 64

FOREIGN LANGUAGE INDICATORS

for beginning a foreign phrase 3.4.3 64

for ending a foreign phrase 3.4.3 64

for one foreign character 3.4.3 64

for one foreign word 3.4.3 64

FRACTIONS

continued 5.7.3 134

discussion regarding 5.7 128

of higher order 5.7.1 129

spatial representation of 5.7.2 130

together with radicals 5.7.7 139

(See also SIMPLE FRACTIONS.)

FRENCH NUMBERS (See DOT-6 NUMBERS.)

FUNCTION ABBREVIATIONS

discussion regarding 5.9 140

list of 5.0.7 117

G

GEOMETRIC MODIFIERS

discussion regarding 5.9.1 141

examples of 5.9.1 142

list of 5.0.8.1 118

GEOMETRIC SHAPES

discussion regarding 5.9.2 142

list of 5.0.8.2 118 6.46

GREEK ALPHABET

discussion regarding 3.9.1 85

list of lower-case letters 3.0.7 39

list of upper-case letters 3.0.8 40

GROUPING SIGNS

discussion regarding 3.7.3 68 and 5.10 143

examples of 5.10 143-146

list of 3.0.9 40 and 5.0.9 119

H

HELP FOR YOUNG CHILDREN

avoiding paper creep 4.3.2 103

column separation 4.3.1 102

HYBRID CONSTRUCTS

discussion regarding 2.3 29

examples of 2.3 29

HYBRID WORD

definition of 2.4 31
 discussion regarding 2.4 31
 examples of 2.4 31

HYPHEN

discussion regarding 3.2.9 53
 example as a non-delimiter 2.3 28

HYPHENATION

discussion regarding 3.8.3 82
 with compound words 3.8.3.3 84
 with prefixes 3.8.3.1 83
 with suffixes 3.8.3.2 84

I

INCHES 3.2.4 51

INDICATORS

list of 3.0.10 41-42 and 5.0.10 120

INTRODUCTION ---- *ii*

ITALIC INDICATORS

for beginning a phrase 3.6 66
 for ending a phrase 3.6 66
 for one character 3.6 66
 for one word 3.6 66

L

LEGAL SIGNS

discussion regarding 3.7.4 70
 list of 3.0.11 42

LEVEL INDICATORS (See SIMPLE SUBSCRIPTS AND SUPERSSCRIPTS.)

LIST OF

accent marks 3.0.1 37
 arrows 3.0.2 37 and 5.0.1 115
 ASCII character set 5.0.2 115
 auxiliary notation 5.0.2.1 116
 comparison signs 3.0.3 38 and 5.0.4 116
 compound words 3.8.3.3 84
 currency signs 3.0.4 38
 delimiters 2.3 27
 diacritics 3.0.1 37
 English lower-case letters 3.0.5 38
 English upper-case letters 3.0.6 38
 functions and their abbreviations 5.0.7 117
 geometric modifiers 5.0.8.1 118
 Greek lower-case letters 3.0.7 39
 Greek upper-case letters 3.0.8 40
 grouping signs 3.0.9 40 and 5.0.9 119
 indicators 3.0.10 41 and 5.0.10 120
 legal signs 3.0.11 42

- logic signs 5.0.11 120
- miscellaneous signs 3.0.12 42
- modifiers 5.0.12 121
- numeric signs 3.0.13 42
- operation signs 3.0.14 43 and 5.0.13 121
- prefixes 3.8.3.1 83
- punctuation 3.0.15 43
- reference signs 3.0.16 44
- related signs for simple fractions 3.0.17 44
- related signs for simple radicals 3.0.18 44
- related signs for simple subscripts and superscripts 3.0.19 45
- set theory signs 5.0.11 120
- special letters 5.0.15 121
- suffixes 3.8.3.2 84
- supported languages 3.4.3 64

LOGIC SIGNS

- list of 5.0.11 120

M

METAPHOR 1.4 15

MINUTES 3.2.5 51

MISCELLANEOUS SIGNS

- discussion regarding 3.7.5 70

- list of 3.0.12 42

MISSION ---- 11

MIXED NUMBERS

- definition of 3.7.8.4 76

- discussion regarding 3.7.8.4 76

- examples of 3.7.8.4 76

- rules regarding 3.7.8.4 76

MNEMONICS FOR NUMBS SYMBOLS

- discussion regarding 1.7k 20

- examples of 1.7k 20

MODIFIERS

- discussion regarding 5.13.7 153

- examples of 5.13.7 154

- list of 5.0.12 121

N

NARRATIVE PHRASE

- definition of 2.5 33

NARRATIVE WORD

- definition of 2.4 30

- examples of 2.4 30

NOTATIONAL INDICATOR

- for an enclosed notational phrase 2.5 34
- for a notational phrase that begins with a digit 2.5 34
- for a notational phrase that begins with a non-digit 2.5 34
- for a notational word that begins with a digit 2.4 32
- for a notational word that begins with a non-digit 2.4 32

NOTATIONAL PHRASE

- definition of 2.5 33
- examples of 2.5 33

NOTATIONAL WORD

- definition of 2.4 30
- examples of 2.4 31

NUBS (See PROPOSED NAME.)

- principles and guidelines for 1.7 18
- scope of 1.6 17

NUMBERS

- Antoine (See dot-6.)
- comma and decimal point in 3.3.5 60
- continental usage in 3.3.6 60
- discussion regarding 3.3 56
- dot-6 3.3.2 58
- dropped 3.3.3 58
- French (See dot-6.)
- Roman 3.3.7 60-61
- that are not subscripts 3.7.10.2 80
- upper 3.3.1 56

NUMERIC INDICATOR 3.3.4 59

NUMERIC SIGNS

- list of 3.0.13 42

NUMERIC SUBSCRIPTS

- discussion regarding 3.7.10.2 80
- examples of 3.7.10.2 80

O

ONE-HALF AND ONE-FOURTH 3.7.8.5 76

OPERATION SIGNS

- discussion regarding 3.7.6 71
- list of 3.0.14 43 and 5.0.13 121

OTHER ALPHABETS 3.9.2 86

P

PARENTHESES 3.2.2 48

PERIOD 3.2.10 54

PHRASES AND PASSAGES

- discussion regarding 2.5 33
- examples of notational 2.5 33
- notational 2.5 33
- rules regarding 2.5 34

PREFACE ---- *i*

PREFIXES

- discussion regarding 3.8.3.1 83
- hyphenation with 3.8.3.1 83
- list of 3.8.3.1 83

PRESERVATION OF STRUCTURE

- discussion regarding 1.7p 21
- examples of 1.7p 21

PRIME 3.2.5 51

PRIMES TOGETHER WITH INDICES 5.13.1 149

PRINCIPLES AND GUIDELINES

- discussion regarding 1.7 18-22
- equal-access principle 1.7q 22
- just-in-time principle 1.7n 21
- list of 1.7 18-22
- minimal-enclosure principle 1.7o 21
- preservation-of-structure principle 1.7p 21

PROPOSED NAME 1.1 14

PUNCTUATION

- discussion regarding 3.2 48
- list of 3.0.15 43
- vertical ellipsis 5.0.4 121

PUNCTUATION INDICATOR

- punctuation requiring 3.2.10 55-56
- representation of 3.2.1 48

Q

QUASI-VERTICAL FRACTIONS

- definition of 3.7.8.1 74
- discussion regarding 3.7.8.1 74
- examples of 3.7.8.1 74

QUESTION MARK 3.2.8 53

R

RADICALS

- indexed 5.7.4 136
- of higher order 5.7.4 137
- spatial representation 5.7.6 138
- (See also SIMPLE RADICALS.)

REFERENCE SIGNS

- discussion regarding 3.7.7 72
- list of 3.0.16 44

ROMAN NUMERALS 3.3.7 60-61

RULES REGARDING

- cancellation 4.0.6 110
- complexity of a fraction 5.7.1 129
- complexity of a radical expression 5.7.5 137

continued fractions 5.7.3 134
 indexed radicals 5.7.4 136
 level indicators 3.7.10 79
 mixed numbers 3.7.8.4 76
 phrases and passages 2.5 34
 representation of a radical expression 3.7.9 77
 slash 3.7.8.3 75
 spatial representation of fractions 5.7.2 130
 spatial representation of radicals 5.7.6 138
 words 2.4 32

S

SECONDS 3.2.4 51

SEMICOLON 3.2.8 53

SEQUENCING

discussion regarding 3.8.2 81

table regarding 3.8.2 82

SET THEORY SYMBOLS

list of 5.0.11 120

SIGNS

arrows 3.7.1 67

comparison 3.7.2 68

currency 3.0.4 38

grouping 3.7.3 68

legal 3.7.4 69

miscellaneous 3.7.5 70

operation 3.7.6 71

reference 3.7.7 72

SIMPLE FRACTIONS

definition of 3.7.8 72

discussion regarding 3.7.8 72

examples of 3.7.8 73

list of related signs for 3.0.17 44

SIMPLE RADICALS

definition of 3.7.9 77

discussion regarding 3.7.9 77

examples of 3.7.9 78

list of related signs for 3.0.18 44

rules regarding 3.7.9 77

SIMPLE SUBSCRIPTS AND SUPERSCRIPTS

definition of 3.7.10 78

discussion regarding 3.7.10 78

examples of 3.7.10 79

list of indicators for 3.0.19 45

rule regarding 3.7.10 79

SINGLE QUOTES 3.2.5 51

SLASH

- and hyphen as non-delimiters 2.3 28
- discussion regarding 3.2.6 52
- examples of other uses 3.7.8.3 75
- in connection with fractions 3.7.8.3 75
- rules regarding 3.7.8.3 75
- uses of 3.7.8.3 75

SPACE

- as a non-delimiter 2.3 27 and 5.6 128
- examples as a non-delimiter 2.3 28 and 5.6 128

SPECIAL LETTERS

- discussion regarding 3.9.3 86
- list of 5.0.15 122

SUBSCRIPTS AND SUPERSCRIPTS (See SIMPLE SUBSCRIPTS AND SUPERSCRIPTS.)

SUFFIXES

- hyphenation with 3.8.3.2 84
- list of 3.8.3.2 84

T

TEACHER ALERT REGARDING

- accent marks 3.4.4 65
- cancellation in subtraction 4.5.1 107
- fractions 3.7.8.2 74
- importance of fonts 2.4.1 32
- level indicators 3.7.10.1 79
- punctuation 3.2.11 56
- shapes of letters 3.1.5 48
- upper-case letters 3.1.5 48

TRIGONOMETRY

- discussions regarding 5.14 155
- list of functions 5.0.16 122

TYPOGRAPHIC CONVENTIONS

- discussion regarding 3.6.1 66

U

UNDERLINE INDICATORS

- for beginning a phrase 3.6 66
- for ending a phrase 3.6 66
- for one character 3.6 66
- for one word 3.6 66

UNDERSCORE 3.2.9 54

UNIFORM 1.2 14

UPPER-CASE INDICATORS

- for beginning an English phrase 3.1.4 47
- for ending an English phrase 3.1.4 47
- for one English letter 3.1.2 46
- for one English word 3.1.3 47

UPPER NUMBERS 3.3.1 56

V

VERTICAL ELLIPSIS 5.0.14 121

W

WORD

- analysis of 2.4 30
- as basic building block 2.2 26
- definition of 2.4 30
- discussion regarding 2.4 30
- hybrid 2.4 31
- intuitive approach to 2.2 27
- narrative 2.4 30
- notational 2.4 30
- types of 2.4 30
- rules regarding 2.4 32