**NUBS Course for Scientific Notation**

This manual is Version 1 provided in 2017. If there are changes and or corrections needed, please contact ABL at the web site from which you received this.

This Course for Scientific Notation covers topics from the less complex to the most complex. The rules presented in the manual for Literary NUBS are retained in this manual; NUBS is a single, complete braille system. The material presented in these sections is related to new symbols, to formatting, and descriptions of how to apply the basic rules to science concepts. There are a number of examples for each topic. Appendices provide lists of all defined symbols and there are also rules available for devising undefined symbols as needed.

It is necessary for you to have completed the Literary Course for NUBS before starting this course about scientific notation using NUBS. We recommend that you review Chapter 14 before you begin this course as the information there is basic to all that this manual presents.

The first section gives a description of how notational phrases are used. Material is presented so that everything needed for a particular subject is in one section. For example, everything needed to handle addition and subtraction, including any special rules for formatting, is in Chapter 2. Chapter 3 covers Multiplication and Division in the same manner.

Topics are covered in ten chapters. Chapters two through eight have one or more drills for practice. Chapters two through seven have Exercises at their ends that should be sent to the instructor for review. After successful completion of an exercise, the student is ready to continue with the following chapter. Chapters nine and ten deal with advanced topics and give examples of how to apply the rules to the numerous symbols used in higher mathematics. There are no drills or exercises for these two chapters. Students should at this point be able to apply what has been learned. If questions arise, the instructor may be contacted for help.

The chapters are arranged so that the less complex topics are described before those that cover higher level subjects. A transcriber need only to study topics up to and through those that pertain to their needs.

**Table of Contents**

Chapter 1 Rules Refresher …………………………………………………………..…….. 1

Notational and Narrative Indicators ……………………………………….…….… 1

Notational Punctuation …………………………………………………………….. 1

Notational Phrases …………………………………………………………………. 1

Linked Expressions …………………………………………………………………2

Mixed Narrative and Notational Statements ………………………………………. 2

Roman Numerals …………………………………………………………………... 2

Ordinal Endings ……………………………………………………………………. 3

Decimals …………………………………………………………………………… 3

Proportions ………………………………………………………………………… 4

Ordered Pairs, Numbers Within Signs of Grouping ………………………………. 4

Division of Long Numbers ………………………………………………………... 4

Units of Measure ………………………………………………………………….. 5

Definite Points of Time …………………………………………………………… 6

Intervals of Time ………………………………………………………………….. 6

Dates ………………………………………………………………………………. 6

Sports Scores and Votes …………………………………………………………... 7

Electronic Addresses and other computer texts ………………………………….. 7

Hyphens and Dashes in Mathematical Expressions ……………………………… 7

Chapter 1 Exercises ………………………………………………………………. 8

Chapter 2 Addition and Subtraction ……………………………………………………… 9

Signs of Operation ……………………………………………………………….. 9

Signs of Comparison …………………………………………………………….. 9

Other Symbols …………………………………………………………………… 9

Linear Format Addition and Subtraction ………………………………………… 9

Examples ……………………………………………………………….. 10

Summary of Rules for Linear Format ………………………………………….. 10

Drill #1 …………………………………………………………………………. 10

Spatial Display Addition and Subtraction ……………………………………… 11

Examples ……………………………………………………………….. 11

Format Issues for Spatial Display ……………………………………………… 11

Drill #2 …………………………………………………………………………. 12

Carried Numbers ……………………………………………………………….. 12

Borrowed Numbers …………………………………………………………….. 13

Special NUBS Technique ……………………………………………… 13

Drill #3 …………………………………………………………………………. 15

Chapter #2 Exercises …………………………………………………………… 15

Chapter 3 Multiplication and Division…………………………………………………. 16

Signs of Operation and Comparison ……………………………………………. 16

Division of Long Mathematical Expressions …………………………………… 16

Linearly displayed Multiplication ………………………………………………. 17

Spatially Displayed Multiplication ……………………………………………… 17

Drill #4 …………………………………………………………………………... 19

Linear Display Division …………………………………………………………. 19

Spatial Display Division …………………………………………………………. 19

Drill #5 …………………………………………………………………………… 21

Chapter 3 Exercises ………………………………………………………………. 21

Chapter 4 Fractions ………………………………………………………………………. 22

Simple Fractions and Mixed Numbers …………………………………………… 22

Vertical and Quasi-vertical Fractions ………………………….………………… 22

Fraction Indicators ……………………………………………………………….. 23

Mixed Numbers ………………………………………………………………….. 23

Spatial Format ……………………………………………………………………. 23

Mixed Numbers in Spatial Format ……………………………………………….. 24

Drill #6 ……………………………………………………………………………. 25

Cancellation in Fractions …………………………………………………………. 25

Rules for Cancellation ……………………………………………………………. 25

Complex Fractions ……………………………………………………………….. 27

Hypercomplex Fractions …………………………………………………………. 28

Higher Order Fractions in Spatial Display ……………………………………….. 29

Drill #7 ……………………………………………………………………………. 31

Chapter 4 Exercises ……………………………………………………………….. 31

Chapter 5 Subscripts and Superscripts ……………………………………………………. 33

Indicators ………………………………………………………………………….. 33

Superscripts ……………………………………………………………………….. 33

Numeric Subscripts ……………………………………………………………….. 34

Use of Subscripts …………………………………………………………………. 34

Complex Subscripts ………………………………………………………………. 35

Legal and Business Symbols ……………………………………………………… 35

Drill #8 ……………………………………………………………………………. 36

Chapter 5 Exercises ……………………………………………………………….. 36

Chapter 6 Radicals (Square Root) ………………………………………………………… 37

Radical Symbols …………………………………………………………………... 37

Linear and Spatial Displays ……………………………………………………….. 38

Indexed and Nested Radicals ……………………………………………………… 38

Indexed Radicals ………………………………………………………………..… 38

Nested Radicals …………………………………………………………………… 39

Spatial Representation of Radicals ………………………………………………... 40

Fractions and Radicals Together in Spatial Representation ………………………. 41

Drill #9 ……………………………………………………………………………. 41

Chapter 6 Exercises ……………………………………………………………….. 42

Chapter 7 Geometry ………………………………………………………………………. 43

Shapes in Geometry ………………………………………………………………. 43

Shading Shapes …………………………………………………………… 44

Shapes in Plane Geometry ………………………………………………... 46

Modifiers ………………………………………………………………………….. 46

General Modifiers ………………………………………………………… 46

Geometry Modifiers ………………………………………………………. 47

Rules for Modification ……………………………………………………. 47

Limits of the Modification Process ……………………………………….. 48

Other Uses of Mathematical Shapes ……………………………………………… 48

Drill #10 …………………………………………………………………………... 50

Chapter 7 Exercises ………………………………………………………………. 50

Chapter 8 Trigonometry ………………………………………………………………….. 51

Functions and their Abbreviations ……………………………………………….. 51

Rules for Functions ………………………………………………………………. 52

Drill #11 ………………………………………………………………………….. 53

Chapter 9 Determinants and Matrices ……………………………………………………. 54

Signs of Grouping ………………………………………………………………… 54

Rules for Determinants and Matrices …………………………………………….. 55

Chapter 10 Calculus and Logic and Set Theory ………………………………………….. 57

Symbols Used …………………………………………………………………….. 57

Overscripts and Underscripts …………………………………………………….. 58

Indicators for Overscripts and Underscripts ……………………………………… 59

The Integral Sign …………………………………………………………………. 60

Superimposed Signs ……………………………………………………………… 60

Logic and Set Theory …………………………………………………………….. 61

Logic and Set Theory Characters ………………………………………………… 61

Appendices ……………………………………………………………………………….. 63

MA. Signs of Operation …………………………………………………………. 63

MB. Signs of Comparison ……………………………………………………….. 63

MC. Fractions and Radicals ……………………………………………………… 64

MD. Geometry Shapes and Modifiers …………………………………………… 65

ME. Trigonometry Functions and Abbreviations ……………………………….. 67

MF. Signs of Grouping ………………………………………………………….. 69

MG. Calculus Symbols …………………………………………………………... 70

MH. Greek Alphabet …………………………………………………………….. 71

MI. Logic and Set Theory ………………………………………………………… 72

MJ. Miscellaneous symbols ……………………………………………………... 73

MK. Braille Answers for Drills ……………………………………………… 75 - 84

**Chapter 1**

**Rules Refresher**

**In General**

Everything you learned in the Literary Course of NUBS applies to the math and science aspects. This is all one braille system. The information here is based on the NUBS Literary Manual, especially Chapter 14; we recommend reviewing chapter 14 before moving ahead in this course.

**Notational and Narrative Indicators**

Numeric Indicator (number sign) # (3456)

Notational Indicator ; (56)

Notational Phrases

Open if phrase begins with a digit ;# (56,3456)

Open if phrase begins with a non-digit ;; (56,56)

Close phrase in both cases ;' (56,3)

**Notational Punctuation:**

Period ] (12456)

Colon : (156)

Mode switch (dot-5 switch) (5)

Punctuation Indicator \_ (456)

Narrative: required before punctuation that could be misinterpreted as a contraction.

Notational: required before punctuation that could be misinterpreted as a digit.

Left Parenthesis Narrative \_( Notational ;(

Right Parenthesis Narrative \_) Notational )

Opening and closing parentheses are clearly identified as such in NUBS so there are no restrictions on what words may be in contact with parentheses. Rules for using parentheses are found in Lessons 3 and 7.2.1 of the NUBS Literary Manual.

**Notational Phrases**

Any math statement of three or more notational words will be placed between the notational phrase indicators. These are listed above. A notational phrase insures that all the words are readily understood as being notational and that spaces between them do not terminate the notational mode.

Equations are to be enclosed within notational phrase indicators since they consist of three or more notational words; this means that regardless of spaces around comparison signs all the elements of the equation will be notational. There is thus no need to place a notational indicator before any of the terms within a notational phrase.

**Examples:**

87¢ +5¢ = 92 ¢ ;#87@c+5@c = 92@c;'

$2.45+.35 = -?- ;;@s2]45+]35 = \;'

$3.00-$.50 = $2.50 ;;@s3][00-@s](mailto:00-@s)]50 = @s2]50;'

Notational phrases may be used for material that is being spatially displayed. This includes any number of examples that are displayed together. To set up a phrase for this type of material, begin with the indicator for a phrase that begins with a digit. This is placed at the start of the blank line required for displayed materials. The close indicator is placed at the start of the blank line that follows the displayed material.

When a notational phrase is used, any problem identifiers for examples may be included within the phrase, but also may optionally be given their own notational indicators. The option chosen must be consistent throughout the transcription, and a transcriber note should describe the choice.

**Linked Expressions**

**A.** Linked expressions should be placed within notational phrases.

B. Signs of comparison should be vertically aligned.

C. If there are expressions to the left of comparison signs after the initial entry, follow print as much as possible. This may require adjusting previous lines to the right to align the comparison signs.

D. If material after a comparison sign is too long to fit on one line, let it run over to the next line, indented two cells from the above comparison sign.

**Mixed Narrative and Notational Statements**

At times a mathematical statement will use words within them. The transcriber must decide where to have the notational part of the statement begin and end. It is best to have the words be in narrative mode to save space.

**Examples:**

D=r×3= 3r =distance traveled by slower plane in 3 hours.

**;;**D = r.[3 = 3r =;'distance traveled by slower plane in #3 hours.

Distance traveled by faster train in 2 hours = r×2=2r

Distance traveled by faster train in #2hours ;;= r.[2 = 2r];'

These examples show that a transcriber may choose how to use notational phrases to good advantage in setting up similar sentences.

**Roman Numerals**

Roman numerals are brailled just as they are shown in print when transcribed. All Roman numerals are notational and a single notational indicator is required before the first letter of the numeral. Follow capitalization shown in print.

**Examples:**  V ;,v XL ;,,xl x ;x iii ;iii

**With Hyphen, Colon or Dash**

When Roman numerals are separated by a **hyphen** or a **colon,** no indicators are repeated after these signs. However, the appropriate notational and capitalization indicators **are repeated** after a **dash**, which is an unconditional delimiter. For readability, these units should not be divided between braille lines.

**Examples:**  VI—X ;,,vi.-;,x VI-X ;,,vi-x

VI:X ;,,VI:X

v—x ;v.-;x v**-**x ;v-x v:x ;v:x

**With Prefixes or Suffixes**

When Roman numerals are preceded or followed by a prefix or suffix, a device is needed to separate the Roman numeral from a prefix or suffix. A dot-5 is used for this purpose; there are other uses for the dot 5 that will be discussed later. In this usage, the dot 5 does not alter the mode. The notational mode is retained. The dot 5 simply serves as a **separator** between the Roman numeral and any prefix or suffix.

If a period precedes a suffix, the dot 5 is not used. The dot-5 or the period in Roman numerals terminates capitalization, so any capitalized prefix or suffix requires its own capital indicator.

**Examples:**  aVII ;a",,vii avii ;a"vii

XXVa ;,,xxv"a xxA ;xx",a VI.A ;,,vi},a

**Ordinal Endings**

A hybrid word is one in which part of the word is narrative and part is notational. The use of hybrid words is described fully in sections 5.5 and 12.4 of the NUBS Literary Manual. It involves using a dot 5 to switch from the notational mode to the narrative mode within one word. There are several uses for this dot-5 switch; one use is described here for ordinal endings.

The dot-5 switch is required in Roman or Arabic numbers that are followed by ordinal endings, st, nd, rd, th. Contractions can be used in English ordinal endings, but not with endings in other languages.

**Examples:** 10th #10"? xth ;x"? 21st #21"/

XXIst ;,,xxi"/ 2nd #2"nd 3rd #3"rd

Ier (French) ;,I"er VIe (French) ;,,vi"e 5ten (German) #5"ten

**Decimals**

Since a decimal point in print is identical to a period, the same is true in NUBS. Note that, if the number starts with a decimal point, a notational indicator must precede it rather than a numeric indicator. This is because a decimal point, though notational, is not a digit.

**Examples: .**7 ;]7 **.**03 ;]03 **.**5 - **.**7 ;]5-]7

When a number consists of a whole number and a decimal part, only one numeric indicator is required, and it is placed before the whole number.

**Examples:**  90**.**9 MHz #90]9 ,m,hz 0**.**5-1**.**2 #0]5-1]2

6-7**.**25 #6-7]25 7**.**25-8 #7]25-8

**Proportions**

A**s** with other signs of comparison, any math statement of three or more notational words will be placed between the notational phrase indicators. Those are listed with other indicators at the beginning of this chapter. These insure that all the words of the phrase are readily understood as being notational.

In mathematics, a structure is needed to indicate that two quantities have a relationship which is the same as that for two other quantities. For example, it is clear that “2 is to 4 as 8 is to 16”; that is, the second quantity is twice the first quantity. A single colon is used for the words "is to" and two colons together are used for the word "as" in a proportion statement. The braille notational colons are used for proportions.

**Examples:** x : y :: 3x : 3y ;;x:y :: 3x:3y;'

x is to y as 3x is to 3y

2 : 4 :: 8 : 16 **;**#2:4 :: 8:16;'

2 is to 4 as 8 is to 16

Note that since the "as" is a sign of comparison, spaces are necessary on each side of it, and because it is within a notational phrase, the spaces do not change the mode.

**Ordered Pairs and numbers within Grouping signs**

Numbers used to indicate coordinates on a graph are shown in parentheses with a comma between them. The notational indicator is needed before the first parenthesis to place the expression into notational mode. The closing parenthesis is already in the notational mode, so it does not need an indicator.

If there is a space after a comma, the mode is automatically changed into the narrative mode, so it is necessary to set the following number or letter into the notational mode. To avoid that need, you may instead set the whole item into a notational phrase so that the spaces do not close the notational mode.

**Examples**: (x,y) ;(x\*y) (a,b) ;(a\*b)

(x, y) ;(x\* ;y) or ;;(x\* y);'

(6,8) ;(6\*8) (12,6) ;(12\*6)

(6, 8) ;(6\* #8) or ;;(6\* 8);'

**Division of Long Numbers**

The rules for dividing long numbers that cannot fit on a single braille line are relatively simple.

1) If the number will fit on a single braille line, do not divide it, even if you must start a new braille line to avoid a runover.

2) If the long number will not fit on a single braille line, start the number on the current braille line.

a) Place any divided expression within a notational phrase; this removes any need to start the additional lines with notational or numeric indicators.

b) If a number needs to be divided between lines, a continuation indicator, dots (456), is placed at the end of each line that has a continuation following it.

c) If a number has commas separating groups of three digits, then the braille line will be ended after the last comma that will fit on the line, allowing for the one cell required continuation indicator. Continue with the rest of the number on the next line or lines and end it with the closing notational phrase indicator.

d) If the long number does not have “grouping” commas, fill the line with as many digits as will fit, reserving the last cell for the continuation indicator and continue the number on the next line. Again, a notational phrase will be used and will be ended after the last digit.

**Examples:**

A decillion is 1,000,000,000,000,000,000,000,000,000,000,000.

,a decillion is ;#1\*000\*000\*000\*000\*000\*000\*\_ 000\*000\*000\*000\*000];'

102030405060708090102030405060708090102030405060708090102030405060708090

;#102030405060708090102030405060708090102030\_ 405060708090102030405060708090;'

**Units of Measure**

For units of measure, do not use the rules for abbreviations and acronyms. Instead use the following rules:

1. If the unit of measure is not in contact with any digits, and if it stands separate from other abbreviations, it is narrative. If two or more abbreviations with periods between them are un-spaced from each other, they are notational. Another special case is noted under number 3 below.

**Examples:** ft**.** ft4 mL m,l hr. hr4

sq**.**  ft**.** sq4 ft4 sq**.** ft**.** ;sq]ft]

cu**.**yd**.** ;cu]yd]

2) If the unit of measure is un-spaced from one or more preceding digits and,

a) if the unit of measure is **notational**, the word is notational.

**Examples:**

2sq.ft. #2sq]ft]

We dug 50cu.yd. of sand. ,we dug #50cu]yd] ( s&4

b) If the unit of measure is **narrative,** the word is **hybrid.** Remember, a hybrid word is one in which part of the word is notational and part is narrative. In a hybrid word that begins in the notational mode, a dot-5 “switch” is required before the narrative part, and the rest of the word is narrative.

**Examples** - hybrid:

5mL #5"m,l 5KW **#5",K,W**

2yds, 4ft**.** #2"yds1 #4"ft4

3) Do not use the *in* contraction when it represents an abbreviation for *inch* (or *ins* for *inches*). Any unit of measure containing this use of *in* is notational. This applies regardless of spacing or punctuation.

**Examples:**

8 ins. #8 ;ins] 3ft, 6in #3"ft1 #6in

4 in #4 ;in 8-in. dia. #8-in] dia4

She is 5'4" tall, so shorten her dress 3ins.

,%e is #5'4'' tall1 s %ort5 h] dress #3ins]

Other words containing the contraction for *in* should be treated according to the general rules for abbreviations.

**Example :** 10 mins. #10 m9s4

**Definite Points of Time**

When a definite point of time appears in print, using digits and colons, a notational colon: (156) is used in braille, to separate the hours, minutes, and seconds. Only one numeric indicator is required. These numerical units cannot be divided between braille lines.

**Examples:**

11:30 p.m. #11:30 ;p]m]

1:00:15 a.m. #1:00:15 ;a]m]

**Intervals of Time**

When intervals of time are expressed by whole numbers, such as 6-7 p.m.,only one numeric indicator is needed as with any other hyphenated numerical expression.

In print, intervals of time are joined either by a short dash or by a hyphen. Only the dash would require a repeat of the numeric indicator.

**Examples:**

3-4 a.m. #3-4 ;a]m] 6-7:12 #6-7:12

4:15-5:00 #4:15-5:00 5:15—6 #5:15.-#6

**Dates**

Follow print when a date is represented by the numbers of the month, day, and year (or day, month, and year) separated by slashes or hyphens. Do not divide between lines. Remember, a slash between digits is not a delimiter.

**Examples:**

5/12/08 #5\_/12\_/08 5-12-08 #5-12-08

5.12.08 #5]12]08

If inclusive dates are joined by a hyphen or a dash, follow these examples. Although a slash following a digit is not a delimiter, the dash is an unconditional delimiter, regardless of mode.

**Examples:**  Date elements separated by slashes, joined either by hyphens or dashes.

Fall Festival 10/2-10/7 ,fall ,fe/ival #10\_/2-10\_/7

Fall Festival 10/2—10/7 ,fall ,fe/ival #10\_/2.-#10\_/7

**Examples:**  Date elements separated by dots or periods, joined either by hyphens or dashes.

Fall Festival 10.2-10.7 ,fall ,fe/ival #10]2-10]7

Fall Festival 10.2—10.7 ,fall ,fe/ival #10]2.-#10]7

**Example:** Date elements separated by hyphens, joined by dashes.

Fall Festival 10-2—10-7 ,fall ,fe/ival #10-2.-#10-7

**Sports Scores and Votes**

Numbers representing sports scores or voting results usually appear in print using hyphens or dashes. These should be brailled to match the print. Note that in the second example, a numeric indicator is required after the dash.

**Examples:** 98-102 12,888—11,045.

#98-102 #12\*888.-#11\*045

**Electronic Addresses and Other Computer Texts**

Computer text, including electronic addresses, is transcribed in the notational mode which provides a representation for each symbol needed in computer text. Of course, no contractions are permitted.

An example of an electronic address is: http://msdn@microsoft22.com

;http:\_/\_/msdn@amicrosoft#22]com

Notice that the numeric indicator is required before any digit or series of digits within the address to avoid being misinterpreted as subscripts.

If a long electronic address or other unbroken computer text is too long to fit on a single braille line, divide it between braille lines using the continuation indicator (456) as shown here.

http://www.angelfire.com/biz/casc/index.html

;http:\_/\_/www]angelfire]com\_/biz\_

\_/casc\_/index]html

**Hyphens and Dashes in Mathematical Expressions**

When two whole numbers are connected by a **hyphen,** only one numeric indicator is required. When two whole numbers or two fractions are connected by a **dash,** an additional numeric or notational indicator is required, since the dash is an unconditional delimiter.

**Examples:**

3-6 #3-6 3—6 #3.-#6 a—b ;a.-;b

 ;?1/2#-?3/4# — ;?1/2#.-#1?1/2#

 ;?1/2#-1  #1?1/2#-1-?1/2#

0 — #0.-;?1/2# 33 3**/**8 #33?3/8#

**Chapter 1 Exercises**

Transcribe each of the following questions. You do not need to solve any of the questions.

1. A cornerstone is marked MCMLXXII. What date does it represent?

2. List the definitions given in Chapters I-V and XI-XV.

3. What value do the XX's have in LXX?

4. Is your son in 2nd or 3rd grade?

5. The fair will be held on the 1st, 2nd, 3rd, and 4th of September.

6. Add .45, .76, .38 together.

7. Find X in the following proportions: 32:64 :: 8:X. 3:18 :: X:36

8. Here are a few examples of ordered pairs. (x,y) (a, b) (3,4)

9. Can you name this number? 123,456,789,012,345

10. 987,654,321,123,456,789,987,654,321,123,456,789 is a huge number.

11. How many feet in 2 yards?

12. 9 sq.ft. is 1 sq. yd.

13. His appointment is set for 2:15 p.m.

14. School will be closed from 11/22–11/25 for Thanksgiving.

15. Our school team won the game 42-34.

16. She won the election by 256 votes, 22,346—22,090

17. The ABL website is at [www.allforbraille.org](http://www.allforbraille.org).

18. There are 6-8 cherries in each bag.

19. For you, there are 2—3 to give away.

20. Monday September 11, (9/2/2001) was a very sad day in the USA.

**Chapter 2**

**Addition and Subtraction**

In this section you will find all the information needed to handle addition and subtraction both linearly and spatially.

**Signs of Operation**

minus - - (36)

plus + + (346)

**Signs of Comparison**

is equal to = = (123456)

is greater than > @> (4,345)

is less than < @< (4,126)

is not equal to ≠ .= (46,123456)

**Other Symbols**

Each of the following symbols, if not enclosed within a notational phrase, must have a notational indicator placed before it to make the "word" which it starts notational.

Dollar sign @s (4,234) Cent sign @c (4,14)

Decimal point ] (12456) (The same as the notational period)

General Omission Symbol \ (1256) (This is used to show a blank line, a dash, or a question mark that is standing alone or with hyphens.)

Equations are to be enclosed in notational phrase indicators since the spaces around the signs of comparison would require extra notational indicators after each space if the equations were not within notational phrases.

**Examples:**

87¢ +5¢ = 92 ¢ ;#87@c+5@c = 92@c;'

$2.45+.35 = -?- ;;@s2]45+]35 = \;'

$3.00-$.50 = $2.50 ;;@s3][00-@s](mailto:00-@s)]50 = @s2]50;'

**Linear Format, Addition and Subtraction**

Many simple addition and subtraction problems are shown in a linear format in print.

**Examples:**

2+3 = 5 ;#2+3 = 5;' 39–26 = 13 ;#39-26 = 13;'

36+25 = 61 ;#36+25 = 61;' 117-99 = 18 ;#117-99 = 18;'

63¢ +21¢ = 84¢ ;#63@c+21@c = 84@c;'

Notice that in each of these examples the numeric phrase indicators are needed before the first number and after the last number, but no indicators are used either before or a after the equal sign.

When a letter or other symbol is the first character of an equation, a double notational indicator is used instead of the notational indicator with a numeric indicator.

**Examples:**

$6.92+$.44 = ? ;;@s6]92+@s]44 = \;'

$8.35-$.56 = $7.79 ;;@s8]35-@s]56 = @s7]79;'

x+y+z = 455 ;;x+y+z = 455;'

y+2y = 3y ;;y+2y = 3y;'

-?-+5 = 12 ;;\+5 = 12;'

**Summary of Rules for Linear Format**

A. A beginning notational phrase indicator ;**#** (56,3456) must precede any digit that is at the start of a math problem consisting of three or more notational words.

B. A beginning notational phrase indicator ;; (56,56) must precede any non-digit that starts a math problem consisting of three or more notational words.

C. A closing notational indicator ;' (56,3) must follow the last word of the phrase.

D. Spaces are required before and after any sign of comparison, such as an equal sign or greater than or less than symbol. These spaces do NOT cause a change out of the notational mode because the problem is within a notational phrase.

E. No spaces are left before or after any sign of operation, in this case plus or minus signs.

F. If there are italics in a print problem, they should be ignored.

G. When a notational phrase is used, any identifier for an example may optionally be included within the phrase. The rules described above apply to the example with the notational phrase indicator being placed either before the identifier or before the example itself.

**Examples:**

23 + 326=349 ;#23+326 = 349;'

10 – 8<6 ;#10-8 @< 6;' (10 minus 8 is less than 6)

1. a+b+6 > a+b ;#1] a+b+6 @> a+b;' (a+b+6 is greater than a+b)

**Notational Comma Use:**

A comma within a number must be in the notational mode. For example, in the number 1,000, if a narrative comma were used, the number would read as 11000 because the narrative comma would be mistaken for the number 1. The notational comma \* (16) is used instead to avoid confusion.

2,567 #2\*567 10,245 #10\*245

**Drill #1**

1. In 1,234,567, the 1, 3, 5, and 7 are odd numbers.

2. Add: 30, 40, 60 and70.

3. What does 15+6+150 equal?

4. If 4+6 = 10 then does 10-6 = 4?

5. $3.50+$3.50 = $7.00

6. 50¢-35¢ = 15¢

7. .7 > .3

8. -.32+.98 = +.66

9. 568+297 = ?

10. Round to the nearest tenth: 9.45, 3.86, 423.55

**Spatial Display for Addition and Subtraction**

Braille is designed by its very nature to be a linear presentation of text. This serves well in normal literary applications, but it leaves students unaware of the actual form used in print for simple arithmetic problems. Braille systems solve this with a format called spatial display.

Spatial display is used for material that is set off, in print, from the rest of the text by blank lines above and below the text. Examples in a non-linear format and materials that involve indentation, margin changes, and/or columns or tables are also handled with spatial display.

Consider the examples that are shown below. In print they are set off as displayed text, and spatial display allows braille readers to have the same information as sighted students. Notational phrase symbols are placed at the beginning of the lines above and below the displayed text.

Note the ruled lines, as shown in the braille below; a dot 5 followed by a series of dots 25.

**Examples:**

20 144 2a+3b

+12 − 23 + a+ b

;#

20 144 2a+3b

+12 3 23 + a+ b

"3333"33333 "3333333

;'

**Format issues for spatial display are as follows:**

A. Above the text, there is a blank line with only an open notational phrase indicator placed at its start. Below the text, there is a blank line with only a close notational phrase indicator placed at its start. If the last line of the text is on the last line of a page, the close notational phrase indicator may be placed at the end of that last line instead on the next line. A blank line should then be left at the top of the next page.

B. Entries must be aligned according to their decimal value or by their letters unless they have been intentionally misaligned as an exercise for the student.

C. Plus or minus signs must appear at least one column of cells to the left of the widest column of numeric symbols which appears in the part of the arrangement above the ruled line.

D. The ruled line must extend one cell beyond the left and right extremes of the problem.

E. Operators and variables within a problem, as in the second example below, must be aligned.

F. When spatial arrangements are placed side by side across a page without identifiers, at least two blank spaces must be left between the end of one ruled line and the beginning of the next.

G. When examples have identifier numbers or letters, they are placed on the same level as the ruled lines. It is not required for identifier numbers to be preceded by either a notational indicator or a numeric indicator because they are part of the notational phrase; optionally, the transcriber may use notational indicators on the identifiers. A note should be entered on the transcriber note page indicating which option is used in the transcription.

H. When spatial arrangements are identified by number or letter and are placed side by side across a page, the ruled line of one spatial arrangement, must be at least two cells distant from the identifier of the neighboring arrangement to its right.

I. When numbers need to be carried in addition or borrowed in subtraction, a carried/borrowed number line (one cell of dots 56 followed by a series of dots 2356) is used above the example to separate it from the carried or borrowed numbers.

**Examples:**

2 2x+3y

1. +3 2**.** + x- y

5 3x+2y

;#

2 2x+3y

+3 + x- y

1] "333 2} "3333333

5 3x+2y

;'

Notice that the identifiers are aligned with the ruled line, separated from it by a single cell.

Notice the alignment of digits and variables. The ruled line, a dot 5 followed by a string of dots 25, extends one cell to the left and one cell to the right of the outermost elements of the problem, including the entry below the ruled line.

**Drill #2**

246 385 5a+6b

1. +79 2. +145 3. +2a+3b

4. 5. 6. 7.

**Carried Numbers in Addition**

In spatial addition problems, the number carried over to the adjacent column on the left is often shown **in reduced size** in print. Digit alignment is essential. See the example below.

121

1 2 1 ;7777 <-- carried number line

662 662

1075 1075

+ 974 + 974

2711 "333333 <-- ruled line

2711

Note that a “carried number line” (dots 56, and a series of dots 2356) is placed between the top element of the addition and the carried numbers.

Note that each of the carried numbers is placed above the column to which it is to be added.

The **carried number line** will also be used as the **borrowed number line** in subtraction as shown below.

**Borrowing (Renaming) Numbers in Subtraction**

It is clear at a glance that the example below does not require any “borrowing” or “renaming” since each of the digits being subtracted is less than the digit above it.

In the problem below, the subtraction can be carried out easily so there is no need for carrying any numbers.

1456 1456

- 123 - 123

"333333

The next example does require some renaming. It can be seen at a glance that 7 cannot be subtracted from 4, so we need to "borrow" a 1 from the 9, changing it to an 8 and replacing the 4 with 14. In print, this is no problem in that the number 14 can be written above the 4 in smaller font size so that the alignment of the columns is maintained. This is shown on the right, in print. As seen below, braille doesn't allow the 14 to align very well over the 4.

;#

8 14 814

1394 1394 ;7777

-267 - 267 1394

- 267

"333333

;'

**NUBS solves this subtraction process by providing a clever technique.**

Fortunately, a two-digit replacement number in a subtraction scheme is always in the range of 10 to 18 inclusive. Therefore, we are able to devise a technique for entering this two-digit number into one cell. We consider the unit's digit of this number as a dropped number, as usual. We now add dot 1 into the same cell to indicate that this is a two-digit number whose ten's place is 1. Thus, the numbers from 10 to 18 would be represented in one cell as follows:

z b l h \ < r ( v

10 11 12 13 14 15 16 17 18

Place a borrowed number line, as described above, to separate the borrowed numbers from the problem. This line should be one cell shorter on each end than the ruled line.

Using this technique, a problem requiring carrying would be brailled as:

;#

8\

;7777 <-- borrowed number line

1394 1394

-267 - 267

1127 "333333

1127

;'

This technique does not require cancellation indicators. For subtraction this is all we need to know for cancellation. Here are a few more examples to help you get comfortable with the technique. Note : It can be useful to use a second level of cancelled numbers as in the second example, or it may be handled with just one level depending on the understanding of the student. Also note that these two-digit numbers look the same as some letters and other symbols; when placed above the borrowed number line they instead represent one of the numbers from 10 through 18.

572 231 1,246

-365 - 89 - 159

;#

1l 1h

6l 2b 3r

;777 ;777 ;77777

572 231 1\*246

-365 - 89 - 159

"33333 "33333 "3333333

207142 1\*087

;'

Remember, if a cell above the carried/borrowed line, contains a dot 4, that cell represents one of the teen numbers. This system really does simplify the demonstration of subtraction problems that require borrowing and renaming.

**Drill #3**

Use carry and borrow numbers where needed.

246 387 1,256

1. -79 2. +146 3. +2,576

4. 5. 6. 7.

**Chapter 2 Exercises**

Practice brailling these examples of displayed addition and subtraction. Use carrying and cancellation notations where needed.

1. $4.77+$3.22 = $7.99

2. .09+.22 = .31

3. 368-255+134 = ?

4. Is the following true? 1.29 +2.55-.65 > 3.25

5. 6. 7. 8.

9. 10. 11. 12.

13. 14. 15. 16.

17. 18. 19. 20.

**Chapter 3**

**Multiplication and Division**

**Signs of Operation**

multiplication asterisk \* \_[ (456,246)

multiplication times cross × .[ (46,246)

multiplication times dot " ,] (6,12456)

divided by ./ (46,34)

slash \_/ (456,34)

begin displayed division o (135)

**Signs of Comparison**

is equal to = = (123456)

is greater than > @> (4,345)

is less than < @< (4,126)

is not equal to ≠ .= (46,123456)

**Division of Long Mathematical Expressions Between Braille Lines**

Rules for dividing long expressions:

1. Start all expressions that are too long to fit on one line on a new line. Those that are short enough to fit on the line on which they start should be completed on that line.

2. Place all divided expressions within a notational phrase. It is not necessary to use a continuation indicator in these expressions.

3. Expressions may be divided between lines in the following places, listed in order of preference: before a sign of comparison, before a sign of operation, before a fraction line, before baseline indicators, between factors that are enclosed in signs of grouping, after the last term of a modified expression, after the termination sign for a radical, and before any of the above that are within a superscript or subscript.

4. When an expression needs to extend over more than two lines, divide the expression such that each line after the first line begins with the same type of symbol whenever possible. For example, if an expression consists of two equal signs and requires three lines regardless of how it is divided, the second and third lines should each begin with an equal sign if possible.

5. Should it not be possible to maintain such consistency, break long expressions as necessary to have the segments be approximately the same length. If one needs to be greatly longer than the others, try to have that be the first segment.

6. When multiple multi-line or single-line expressions follow each other directly, indent each runover segment by two spaces from the beginning of the first line of the expression to which it belongs. In a long expression surrounded by text, runover segments may be started directly under the beginning of the first line of the expression if so desired.

**Example:** (235x+456+987y)-(123x+432+675y)+(26x-200-238y)

;;(235x+456+987y)-(123x+432+675y)

+26x-200-238y);'

**Linearly Displayed Multiplication**

Simple multiplication problems can be presented linearly as the ones studied in the NUBS Literary Course. Keep in mind that with the notational phrase indicators, the whole mathematical statement is notational so we do not use additional notational indicators in the middle of the statement.

**Examples:**

3 × 2 × 5 = 30 ;#3.[2.[5 = 30;'

(5a×3)+(6b×2) = 15a+12b ;;(5a.[3)+(6b.[2) = 15a+12b;'

**Spatial Display Format for Multiplication**

Often math problems are displayed in print to make it easier to handle large numbers. This type of problem is handled with Spatial Display in braille.

It is important to note that spatial format does not use notational or numeric indicators for spatial material that is entirely notational. We place an opening notational phrase indicator at the start of the blank line required before the displayed material. We also place a closing notational phrase indicator at the beginning of the blank line that follows the displayed material. Everything between these two indicators is notational.

The ruled line between a problem and its answer, separation line, is composed of a dot 5, followed by a series of dots 25, the same as used in addition and subtraction.

**Format issues for spatial display materials are as follows:**

1. A begin notational phrase indicator ;**#** (56, 3456) must be placed at the start of the blank line required before a spatial display.

2. An end notational phrase indicator ;' (56,3) must be placed at the start of the blank line required after a spatial display. If the last line of the example is on the last line of a page, the end notational phrase indicator may be placed at the end of that last line. A blank line should then be left at the top of the next page.

3. Entries must be aligned according to their decimal value or by their letters unless they have been intentionally misaligned as an exercise for the student.

4. The ruled line separation line must extend one cell beyond the left and right extremes of each example.

5. Operators and variables within a problem must be aligned.

6. When examples have identifier numbers or letters, they are placed on the same level as the ruled lines. One blank space must be leftbetween the last symbol in the identifier and the symbol furthest left in the overall arrangement of the example including its separation lines. Since identifiers are inside the notational phrase they do not need notational indicators. Optionally the identifiers may be provided their own notational indicators.

7. When spatial arrangements are placed side by side across a page without identifiers, at least two blank spaces must be left between every two separation lines.

8. When spatial arrangements are identified by number or letter and are placed side by side across a page, the ruled line of one spatial arrangement, must be at least two cells distant from the identifier of the neighboring arrangement to its right.

9. In a spatial arrangement with multiplication, the multiplier and multiplicand must be aligned as in print. Dollar signs, commas, and decimal points should be placed as in print. The multiplication sign, if shown in print, is placed immediately before the multiplier. The separation line must extend one cell to the left and the right of the longest line above or below it. Additional separation lines must be the same length.

10. If the multiplication contains fractions, mixed numbers, or polynomials, its terms and indicators must be aligned vertically.

11. When a multiplication answer contains a comma or decimal point, the cells in the partial products above it should be left blank.

12. To show carried numbers, a carried number line, as used in addition, may be used to separate the carried numbers from the multiplicand; it consists of dots (56) followed by a line of dots (3456).

**Examples of structured format in multiplication problems:**

;#

101 101

x 22 .[22

202 "33333

202 202

2222 202

"33333

2222

;'

Note that in braille the multiplication sign is positioned un-spaced immediately before the multiplier.

**Example:** (showing column separation to keep digits aligned in the presence of decimal points.)

;#

345.7 345}7 22. x 2.77 .[2}77

24199 22] "33333333

24199 24 199

6914 241 99

957**.**589 691 4

"33333333

957}589

;'

In this example, we know that the answer will have 3 digits after the decimal point because the two multipliers have a combined total of 3 digits after their decimal points.

Notice the alignment of the identifier number (22.) with the top ruled line and the positioning of the problem so its left-most element (the ruled line in this case) is one cell to the right of the identifier number. The identifier does not need a numeric indicator because it is within the notational phrase; it may have a numeric indicator if the transcriber prefers to add it.

**Drill #4**

Braille the following. Show carried numbers in number 6 as an example of how carried numbers in multiplication might look.

1. 12×3×10 = 360 2. 25"3 = ? 3. True or false. 33\*3 < 100

4. 326 5. 189 6. 746

×35 ×31 ×212

1630 189 1492

978 567 746

11,410 5,859 1492

158,152

7. 2.38 8. 78.2 9. 2837

×1.7 ×65.3 ×416

1666 2346 17022

238 3910 2837

4.046 4692 11348

5106.46 1,180,192

**Linearly Displayed Division**

Some division problems are written out in a linear format. These will be done easily with an equation written out on a line. Here are the division symbols for your review.

divided by ./ (46,34)

slash \_/ (456,34)

begin displayed division o (135)

When an identifier is placed one space before an equation it optionally may be included within the notational phrase if desired for the equation; if included, the open notational phrase indicator is placed before the identifier.

**Examples:**

1. 4÷2 = 8÷4 ;#1] 4./2 = 8./4;' or

#1] ;#4./2 = 8./4;'

2. 25÷5 = 5 ;#2] 25./5 = 5;' or

#2] 2#25./5 = 5;'

3. 36÷? = 4 ;#3] 36./\ = 4;' or

#3] ;#36./\ = 4;'

**Spatial Display Format for Division**

Most of the problems involving division do not use the linear format and are presented in the “long division” style in print. We use spatial display for those examples.

**Format for Spatial Display Division**

1. The division symbol o (dots 135) is placed after the divisor and before the dividend.

2. The identifier, if present in print, must be placed on the line with the divisor and is followed by one blank cell. It does not require a notational indicator because it is within the spatial display. As in other cases, a notational indicator may be provided to the identifier if desired.

3. The ruled line (dot 5 followed by a series of dots 25) starts above the division symbol and extends one cell beyond the extreme right of the computation.

4. The structure must maintain columns of equal place value.

5. Other ruled lines are used if they are shown in print. The ends of each should line up with the initial ruled line’s beginning and end.

6. When commas and decimal points occur in a dividend, a column of blank cells should be left below these.

7. A caret, ,5 (6,26), is used to show a new placement for a decimal point; since a caret takes two cells, a two-cell wide column of blank cells must be left below the caret, and the decimal point in the quotient is placed above the right-hand cell of the caret.

8. Remainders: After the quotient and a blank space, a capital "R" indicates that a remainder follows. A numeric indicator followed by the remainder is placed after the "R". The numeric indicator assures that the following number is not mistaken as a subscript.

9. It may be necessary from time to time to allow a division display to extend past the end of a page. This may be done after any of the ruled lines; this may be used to avoid multi-line blank spaces.

Look over the division problem below.

;#

121 ,R#34

121 R34 "33333333333

2**.**  39) 4753 2] 39o4753

39 39

85 85

78 78

73 73

39 39

34 34

;'

1. The divisor (39) and the dividend (4753) are separated by o (dots 135), and the identifier number (2) is placed on level with the divisor and is followed by one blank cell.

2. The structure maintains columns of equal place value.

3. The remainder, if any, follows the quotient and one blank cell above the ruled line. Notice that the R34 that represents the remainder in this problem must include a numeric indicator to show that the 34 is not a subscript on R.

4. The ruled line begins above the division symbol, o dots (135) and extends one cell beyond the extreme right of the computation.

**Drill #5**

Braille the following examples to practice the rules for division in NUBS.

1. 1755÷15 = 117 2. What is 21÷7?

3. 112÷7 = ? 4. 985÷? = 197

5. ?÷21 = 13 6. 36×4÷6 = 24

19 32 8 R 4

7. 23) 437 8. 18) 576 9. 79) 636

23 54 632

207 36 4

207 36

0 0

7.47 13 .7 193 R 15

10. 65) 485.55 11. 2.4^)32.8^8 12. 64) 12367

455 24 64

305 88 596

260 72 576

455 168 207

455 168 192

0 0 15

**Chapter 3 Exercises**

1. 12×? = 144

2. 51,858 ÷ ? = 402

3. 3a×4b = 12ab

4. ab×cd = bd

5. 15z÷3z = 5

6. 345 7. 289 8. 921 9. 46.3 10. 197

×22 ×137 × 38 ×5.69 ×76

1182

1379

14,972

11. 45)9238 12. 44)42271 13. 36)178

14. 11 R44 15. 2.68

63) 737 46 )123.28

63 92

107 31 2

63 27 6

44 3 68

3 68

**Chapter #4**

**Fractions**

**Simple Fractions and Mixed Numbers**

**In-Line Fractions (Fractions printed on the same level)**

When a slash occurs between numbers or letters all printed **on the same level,** it may be a date or an in-line fraction. Such a word is notational, and the standard two-cell slash \_/ (456,34) is used. This format **does not** **apply to in-line** **mixed** numbers which are discussed later.

**Examples:**

3**/**4 lb of butter #3\_/4 lb ( butt]

He wrote 15**/**30 on the list. ,he wrote #15\_/30 on ! li/4

12**/**25 is Christmas Day. #12\_/25 is ,\*ri/mas ,"d4

**Vertical and Quasi-Vertical Fractions**

Fractions that show the numerator and denominator on different levels in print such as 1/3, may have a slanted fraction bar or a horizontal bar in print. Both of these are considered regular fractions. They are treated notationally and require some additional symbols as shown below. The fraction with the slanted bar is called a **quasi-vertical fraction**and the one with the horizontal bar is a standard **vertical fraction***.* These forms of a fraction, where the numerator and denominator are on different levels in print, are both brailled as vertical fractions.

**Fraction Indicators**

Begin fraction ? (1456)

Fraction bar / (34) (Not a two-cell slash !)

End fraction # (3456)

**Examples:**

 ;?1/3#  ;?m/h#  ;?11/16#

Notice that when the first element of the expression is a begin-fraction indicator, a notational indicator (56), rather than the numeric indicator (3456), is required to establish that the word is notational.

**Mixed Numbers**

All mixed numbers are brailled using the same format, even if the fractional part of the number is printed all on one level. A mixed number beginning with a digit uses the numeric indicator to set the notational mode; a mixed number beginning with a variable letter uses the notational indicator.

**Examples:**

   33 1/3

#1?7/8# #4?1/2# ;x?a/b# #33?1/3#

For those who have experience with Nemeth code, you will find much of work with fractions very familiar.

**Simple Fractions in Spatial Format**

As with the spatial display examples in the previous volumes, one or more spatial examples will be preceded by a notational phrase indicator placed at the beginning of the blank line required before the spatial material, that follows the spatial material. There is no need for a notational indicator at the start of a spatial fraction. There is, however, a need for a numeric indicator if a numerator or denominator does not contain any dots (1 or 4) or a notational indicator if a numerator or denominator does not contain any dots (3 or 6). When either dots 1 and 4 or dots 3 and 6, in a numerator or denominator are missing, a braille reader will have difficulty determining the position of the dots without an indicator. This means that a string of digits without any characters that contain a dot 1 or dot 4 will need a numeric indicator, and a group of letters from a to j without other characters that contain a dot 3 or a dot 6 will need a notational indicator.

The fraction bar is formed by the open fraction indicator followed by a series of dots 25 and then the close fraction indicator. The fraction bar will be just long enough for the longer numerator or denominator to fit between the beginning and ending indicators. Both numerator and denominator, even if they are of different lengths, will begin at the left end of the dots 25.

The braille below could be shown together in one phrase if no print material was mixed with them.

**Examples:**

;# ;# ;# ;#

#1 #3 ;c x+y

?33# ?33# ?33# ?333#

#5 #4 ;d m

;' ;' ;' ;'

Note that the last previous example does not need any notational indicator because a dot 1 or a dot 4 and a dot 3 or a dot 6 is present in both numerator and denominator.

;#

#65

= 13 ?333# = 13

#**5**

;'

Note that with numbers, if there are more than one digit in a numerator or denominator, the indicator before the first digit covers the need for positioning for all the digits.

**Mixed Numbers in Spatial Format**

Mixed number fractions are shown with the whole number part on the same line as the fraction bar. Neither a numeric nor a notational indicator is used before the whole number part of the mixed number placed above and below the problem or group of problems. Also, the open fraction indicator provides positioning information for the whole number part; indicators are still used in the numerator or denominator if they are composed only of digits or only of letters from a to j.

**Examples:**

;# ;# ;#

#**5** #15 ;d

3 3?33# 81 81?333# a a?33#

#8 #16 x

;' ;' ;'

**Drill #6**

Practice with a few fractions here. Keep in mind that notational phrase indicators are required for equations.

1. 2. 3. + - = 4. ×

5. ( a + b) 6. $ 3.00 × = $.60 7. =

Practice brailling the following examples in spatial format.

8. 9. 10. 11.

12. 26 13. 7 14. 15. 12y

**Cancellation in Fractions**

Although subtraction schemes do not require cancellation indicators in NUBS, there are some situations where a technique to indicate cancellation is needed. Cancellation is used in the process of reducing fractions to lowest terms, and in the multiplication or division of fractions. In algebra, cancellation is used for the same reasons; however, there we are dealing with number-and-letter combinations rather than just with numbers in arithmetic. NUBS offers a mechanism for showing cancellation in which the extent of the cancellation is precise and the cancelled material remains readable. The cancellation indicators are:

Begin cancellation & (12346)

End cancellation [ (246)

One-character cancellation \_ (456)

**Rules for Cancellation**

1. Text in which cancellation occurs must be presented in spatial format.

2. Text containing cancellation is spatial and must be presented within a notational phrase. The notational phrase indicators will be placed at the start of the blank lines required by spatial display.

3. If the text to be cancelled occupies just one character, the one-character cancellation indicator must be placed above or below it, as appropriate. If the text to be cancelled occupies more than one cell, the begin-cancellation indicator must be placed above or below the first cell of the text and the end-cancellation indicator will be placed above or below the last cell of the text to be cancelled.

**Example:** ;#

#1 #1

1 1 \_ \_

1. × = #3 #4 #1

2 3 1] ?33#.[?33# = ?33#

#8 #9 #6

\_ \_

#2 #3

;'

This example shows how two fractions are multiplied. The 3 in the first numerator cancels with the 9 in the second denominator. These cancelled numbers are replaced respectively by 1 and 3. The number 8 in the first denominator and the 4 in the second numerator have a common factor of 4. These numbers are cancelled and replaced by the result obtained by dividing each by 4. The result is obtained by multiplying the two new numerators and the two new denominators. 1 by 1 and 2 by 3 giving the fraction .

2. (x-a1)(x-a2)(x-a3)(x-a4) = 0

;#

& [

2] (x-a1)(x-a2)(x-a3)(x-a4) = 0

;'

Example 2 shows the product of four algebraic expressions in which the third factor has been cancelled without a replacement.

;#

22 #22

1 88 & [

3. #1 #88

4 1 &[ & [

1 15.[5280

#3] ?33333333#

60.[60

&[ &[

#4 #1

\_

#1

;'

Example 3 above shows cancellation within a fraction that needed two cancellations in a numerator and two in a denominator.

**Complex fractions:**

A complex fraction is one that has a fraction whose numerator or denominator (or both) either are fractions, or contain fractions. We recall that an in-line fraction, consisting of a number followed by a slash and another number all on the same level, is not treated as a fraction. Thus, if both numerator and denominator of what looks like a complex fraction are in-line fractions, then it is actually a simple fraction and does not need the complex fraction symbols.

**Example of an in-line fraction within a fraction:**

;?2\_/3/3\_/2#

**Symbols used in complex fractions:**

Open complex fraction ,? (6,1456)

Complex fraction line ,/ (6,34)

Close complex fraction ,# (6,3456)

The dot 6 used in these symbols is referred to as a complexity indicator. A simple fraction does not use this indicator. A complex fraction uses just one indicator and is also referred to as a first order complex fraction. As fractions increase in complexity, more indicators will be used to signify their level.

Fractions that make up a numerator or denominator are brailled as simple fractions. A complex fraction in NUBS begins with a notational indicator followed by the **open complex fraction** symbol. The numerator is entered as a simple fraction or character. The **complex fraction line** symbol is entered followed by the denominator entered as a simple fraction or character. The **close complex fraction** symbol is entered to complete the complex fraction.

**Examples:**

Complex fraction with numerator as a simple fraction and a digit as the denominator:



;,??3/8#,/5,#

Complex fraction with an in-line fraction as the numerator and a mixed number for the denominator:

 ;,?1\_/2,/2?2/3#,#

Complex fraction with both numerator and denominator within parentheses:



;,?(?1+X/1-X#),/(?1+X/1-x#),#

Simple fraction with both numerator and denominator made up of two in- line fractions:

;?1\_/3+1\_/4/4\_/5-1\_/2#

Complex fraction with both numerator and denominator made up of two simple fractions:

;,??1/2#+?1/3#,/?3/2#-?7/9#,#

Simple fraction with a complex fraction as a superscript or subscript:

+1 ;?a^,??3/4#,/?5/6#,#"+1/b#

b

**Hypercomplex Fractions**

A hypercomplex fraction is one in which the numerator and/or the denominator contain at least one complex fraction.

**Additional symbols needed for hypercomplex fractions**

Begin outermost fraction ,,? (6,6,1456)

Outermost fraction bar ,,/ (6,6,34)

End outermost fraction ,,# (6,6,3456)

Hypercomplex fractions are of the second order of complexity; thus, there are two dot 6 indicators used for the highest level which may also be referred to as the major level.

**Examples:**



1. #1] ;,,?,??1/x#,/y+a,#,,/b,,#

****

2**.** #2] ;,,?,?1?1/4#,/1?3/5#,#,,/5,,#



3.

#3] ;;,,?,?(1-x)?d/dx#(2x)-2x?d/dx#(1-x),/(1-x)^2",#

,,/1+(?2x/1-x#)^2",,#;'

Note the need to return to baseline after the superscripts. Note the use of a notational phrase for the divided example in number 3.

**Higher-Order Fractions and Spatial Format**

The more complex a fraction becomes, the more difficult it is to read in the linear form. Consider the optional braille versions of these hypercomplex fractions which can provide a more readable format. There are two other basic options for displaying the example above. These are either all spatial or part spatial and part linear.In spatial format example, identifying numbers are placed at the start of the line for the major fraction level.

Let’s examine these issues for each of the above examples.

**Example 1 – all spatial**

;#

 #1

?33# lowest level bar

1. x

?3333#

y+a

#1] ;?333333# highest or major level bar

;b

;'

The easiest way to build this example is as follows:

1) Create the simple fraction as an independent expression, starting near the middle of a braille line. Be sure to left adjust the denominator.

2) Under that denominator, create the intermediate level fraction bar, starting it to extend one cell before and one cell beyond the fraction bar above.

3) Place the denominator for that level, left adjusted, below that fraction bar.

4) On the next line, start with the identifying number and then create the major level fraction bar, extending it one cell before and one cell after the previous fraction bar.

5) Place the major denominator, left adjusted, below that fraction bar.

6) Reposition the entire display to its proper placement on the page.

The format below shows the actual physical structure of the print text (except for left adjusted denominators). A less cumbersome option is a mix of spatial and linear formats as follows:



,??1/X#,/Y+A,#

**Example 1** #1] ?33333333333333# ;b

 The format below treats everything above the major fraction bar as a complex fraction using the linear format. This is often the preferred format in that is takes up fewer braille lines and is easier to read. Note that the major denominator is left-adjusted under the brailled fraction line. This use of spatial format for fractions provides a structure that is compact (only 3 braille lines) and yet is easy to read.

,?1?1/4#,/1?3/5#,#

**Example 2** #2] ;?333333333333333333#

#5

Here again, this format is easier to read and involves only a complex fraction as the numerator in a displayed format and a single digit as the denominator. Again, notice the required notational indicator or a numeric indicator preceding the denominators; each, standing alone, are missing either an upper dot, 1 or 4, or a lower dot, 3 or 6. The indicators make it easier to know whether a digit or a letter is there.

Another form provides a separate braille line for the beginning of each fraction level. This representation is entirely linear, difficult to read, but requires only three braille lines. Because it is on separate lines, it must be placed within a notational phrase.

**Example 3**



#3] ;;,,?,?(1-x)?d/dx#(2x)-2x?d/dx#(1-x)

,/(1-x)^2",#

,,/1+(?2x/1-x#)^2",,#;'

Yet another representation is a “first level” spatial – using only the major fraction structure base. Both the numerator and the denominator of the major fraction are shown linearly.

,?(1-x)?d/dx#(2x)-2x?d/dx#(1-x)

,/(1-x)^2",#

#3] ?3333333333333333333333333333333#

1+(?2x/1-x#)^2

If one additional “layer” of spatial structure is used, the following format results.

;#

;d ;d

(1-x)?33#(2x)-2x?33#(1-x) dx dx

?3333333333333333333333333# (1-x)^2

#3] ?333333333333333333333333333# 2x 1+(?333#)^2 1-x

;'

This is probably the most readable, especially for blind students in their first exposure to such complex fractions. It does require five additional braille lines, but readers may find that is a fair trade in the learning process.

**Drill #7**

Practice brailling complex fractions, numbers 1-6 in both the linear and the spatial forms. Use spatial for number 7.

1. 2. 3. 2

4

4. ()×)×) 5. 6. a +

1×2×3 a -

7. (x-1)() + (x-1)() = (x-1)9

**Chapter 4 Exercises**

1. × = = 2. × = = 3. × = =1

1 6 4

4. × = × = 5. × = = 24

5 1 1

6. + = + = + =

7. - = - = - =

8**.** 9**.** 10. 11. **1**

×  **+ +** ×

12 . 13.  **+** 14. 15. 1/3 + 1/4

**-**  4/5 – 1/2

16. 2 17. 33 18. a

4 100

19. x + 2 + 20.

x + 6 +

**Chapter #5**

**Subscripts and Superscripts**

**Simple Superscripts and Subscripts**

Superscript or subscript letters and numerals are used frequently as a student studies algebra and chemistry. Superscripts appear in print as small numbers or letters raised above and to the right of numbers or letters; they indicate a number of times that the number or letter is multiplied by itself. Subscripts appear in print as small numbers or letters to the right of and below a letter. They are used in some advanced mathematics courses and in chemistry.

There are three braille indicators used to transcribe such numbers.

**Superscript and Subscript Indicators**

Superscript ^ (45)

Subscript ; (56)

Baseline " (5)

**Superscripts**

For those of you who are not familiar with the concept of superscripts, they are used as follows. The number 5 with a superscript of 3 means 5 times 5 times 5 and is written as 53 with small 3 above it to the right. It would be brailled as #5^3 using the superscript indicator shown above. In the second of the following examples note the use of the baseline indicator to return to the baseline level after the a2 and the c2.

**Examples:**

The speed of light is 3x108 meters per second.

,! spe$ ( li<t is #3.[10^8 met]s p] second4

a2 + b2 = c2 ;;a^2"+b^2 = c^2";'

After the notational phrase indicator in the second example, we show the "a", followed by the superscript indicator. After the exponent 2, we must use the baseline indicator to show that we are returning to the baseline and that the **+**b**2** is not part of the exponent on "a". The b2 is returned to the baseline by the space that follows it. There is also a dot 5 after the c2 before the closing notational phrase indicator. The notational phrase indicators maintain the notational mode through the spaces around the equal sign.

Look at the next two expressions to see the significance of the **baseline indicator**. The exponent on "a" is different in each example.

a2x+3+4b3 ;a^2x"+3+4b^3 Exponent on *a* is 2*x*.

a2x+3+4b3 ;a^2x+3"+4b^3 Exponent on *a* is 2*x*+3.

The exponent consists of everything that is after the superscript indicator and before either a baseline indicator or a space.

**Numeric Subscripts**

NUBS provides an abbreviated notation for numeric subscripts. This method dispenses with the subscript indicator, and the reader is returned to the base level when the number has ended. To qualify well, this notational abbreviation is available only when a number is a subscript to a letter; it does not apply to numbers which are subscript to other symbols. The letter to which a number is subscript may be in any case, and in any font, or from any alphabet. However, the letter to which the number is a subscript must be at the base level.

**Examples** (of numeric subscripts)

1 x1+x2+x3 ;x1+x2+x3

2 a11x1+a12x2+ ... +a1nxn = b1

;;a11x1+a12x2+ ,,' +a;1n"x;n = b1;'

3 an1x1+an2x2+ ... +annxn = bn

;;a;n1"x1+a;n2"x2+ ,'' +a;nn"x;n = b;n;'

(The numbers which follow **x** are subscripts to **x**, but the numbers which follow **n** are

at the same level as **n**.)

Numbers which follow a letter are not always subscript to that letter. This is particularly true in the case of model and serial numbers and in the identifiers in most programming languages. When the letter and the number are at the base level, the number, in NUBS, must be preceded by the numeric indicator, #.

**Example** (of numbers which follow letters but which are not subscripts)

Vitamin B12 ,vitam9 ;,b#12 (The number follows the letter at the base level.)

**Uses of** **Subscripts**

A **subscript** is often used to identify items in a series, such as A1, A2, A3. In braille any **digits** that directly follow a **letter**are, by definition, a **subscript** as shown in the examples below.

In these cases, no subscript indicator is used so there is no need for the baseline indicator following the subscript. Return-to-baseline is automatic following such an “automatic” subscript.

**Examples:**

a1 ;a1 b2 ;b2 H2O ;,h2,o

CO2 ;,C,O2 CaCO3 ;,ca,c,o3

a1+a2 = n1+n2 ;;a1+a2 = n1+n2;'

There is a rule, learned in the Literary Manual sections 12.4.4 and 12.4.6, that where three or more letters are capitalized within one notational word, the notational word indicator, ,, (6,6), should be used if there are not any lower case letters in the word. Numbers that appear in these formulas are not affected by this use of capitals and the capitalization continues through to the end of the notational word. Chemical formulas, however, are an exception to this rule; in chemical formulas all capital letters must be individually capitalized. If a capitalized notational word is within a notational phrase, the capitalization will stop at a space. Thus, any other words in the phrase will continue to be notational, but capitalization indicators must be applied to other capital letters according to the print material.

Also, recall that if a number following a letter is not a subscript, the numeric indicator must be placed before that number.

**Examples:**

RCV90 ;,,rcv#90

Never confuse H2SO4 with H2O.

,n"e 3fuse ;,h2,s,o4 ) ;,h2,o]

CH3COOH ;,c,h3,c,o,o,h

NaHSO4 ;,na,h,s,o4

NaHSO4+H2O ;,na,h,s,o4+,h2,o

**Complex Subscripts:**

Complex subscripts or subscripts that are not numeric, require the subscript and baseline indicators as shown in the following examples. Since the subscript is not automatically implied as with the numbers shown above, the indicators are necessary. The baseline indicator is also necessary before the closing notational phrase indicator when the last element of an example is a superscript or a non-numeric subscript.

**Examples:**

y = xn+xn-1  ;;y = x;n"+x;n-1";'

a = xn+ xn-1 +3 ;;a = x;n"+x;n-1"+3;'

When digits follow letters and are NOT subscripts, a numeric indicator is required before the digits to indicate that the digit is at the baseline, and is not a subscript.

**Examples:**

K2 is a mountain in the Alps.

;,k#2 is a m.ta9 9 ! ,alps4

The item number is: CM458.

,! Item numb] is3 ;,c,m#458]

**Legal and Business symbols**

Special symbols (often elevated in print) are brailled before or after the word, in accordance with their placement and spacing in print. There is no superscript indicator required when the elevated position is inherent in the braille symbol. The list below shows a few business and legal symbols that have special braille symbols that do not require a superscript indicator. They are, however, notational so they will need a notational indicator placed before them.

Copyright © @.c (4,46,14)

Credit CR  .c (46,14)

Prescription R/ ..r (46,46,1235)

Registered ® @.r (4,46,1235)

Trademark TM @.t (4,46,2345)

**Examples:**

“A Quick-Fix Bar® and lots of H2O is our favorite snack,” said the crew member of America3.

,8,a ,qk-,fix ;,bar@.r & lots ( ;,h2,o is \r favorite snack1,0 sd ! crew memb] ( ;,america^3"]

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;@.c2011 0,! ,^w ,am;g ,u ,press

SX-14TM ;,s,x-14@.t QUAKER® ;,,quaker@.r

The raised number 3 following the word **America** is part of the name of the ship. It is not a true exponent or a reference in the normal use of such terms. A superscript indicator is used to show its position in print and the baseline indicator is used to return to the baseline before the period. The ship is called **America Cubed.**

Notice the notational indicator before the copyright symbol.

The last two examples are fully notational so the one indicator at their beginnings is all that is needed.

**Drill #8**

1. The Pythagorean Theorem is .

2. Einstein's famous equation is E = mc2.

3. y = 9+x4-6x3+18x+3x2

4. NH4 is ammonium.

5. C6H12O6 is glucose.

6. C12H22O11 is sucrose.

7. Evaluate: 2\*82-22\*8 (Use spatial display)

2\*8

8. Copyright © 2008 by The McGraw-Hill Companies

9. Evaluate if r = 2, s = 3, and t =11, s2+(r3-8)+5, 3r(r+s)2-1, and t2+8st+r2

**Chapter 5 Exercises**

1. ax2

2. A56

3. X5.3

4. CO3

5. Na3

6. TE12

7. y-2

8. 2m3+2m3

9. 4(x-y)3-2(x-y)3

10. 3a3+6a6b2+9a9b3

11. (x1y1+x2y2)

12. 8x2+8x-15

13. ([CH3]2 CH)

14. 12ft212ft2 = 144ft2

15. 6x2a +11xa+4

16. yn+yn-1 = z

**Chapter #6**

**Radicals (Square Root)**

**Simple Radical (Square Root)**

As a student progresses through mathematical courses, he or she will find the concept of the square root to be helpful in solving many problems. The print symbol used for square root is  and it is called a “radical”. The braille symbol for square root is made up of a begin radical symbol and an end radical symbol. A radical is used to represent what can be viewed as the inverse of “squaring”. For example, if 4 squared is 16, then the square root of 16 is 4.

**Radical Symbols (Square Root)**

Begin radical > (345)

End radical [ (246)

These symbols are notational so when a mathematical statement begins with a radical sign, it must be preceded by a notational indicator. If a number precedes the radical sign, the numeric indicator is required to set the notational mode for the rest of the statement. Note that the equation examples below are within notational phrase indicators.

**Examples:**

**** = 8 ;;>64[ = 8 ;'

 **;>12]439[**   **;> a+b[**

) **;>s(s-a)(s-b)(s-c)[**

**+5 = (2+5) ;#2>2[+5>2[ = (2+5)>2[;'**

The above examples could be presented as a group within a notational phrase as shown below.

;#

>64[ = 8

>12]439[

>a+b[

>s(s-a)(s-b)(s-c)[

2>2[+5>2[ = (2+5)>2[

;'

Fractions and radicals are used together in many math statements. They may be handled either in a linear format or in a spatial format. The example below shows both ways of brailling such an example.

X = 2 -3

3 +2

Linear: ;;x = ?2>3[-3>2[/3>2[+2>3[#;'

Spatial: ;#

2>3[-3>2[

x = ?333333333#

3>2[+2>3[

;'

Although the linear form takes fewer lines, the spatial form makes it much easier for young readers to follow the structure of the expression.

**Indexed and Nested Radicals**

Radicals can sometimes have a more complex structure than those just shown, such as:

1) Indexed radical: displaying an index such as a 2 for square root or a 3 for cubed root

2) Nested radicals: having a radical occur inside another radical

**Indexed Radicals**

 A radical is used to determine the **root** of a number or of an expression. In the radicals we have studied so far, that root is assumed to be 2 when no other indication is provided. When the cube root of a number is sought, the radical sign for the cube root of 27 would look like this :

This indicates that we are looking for a number "a" which, when multiplied by itself twice, will yield 27. a × a × a yields 27

The answer is 3 and the expression would look like this in braille.

;;>^3"27[ = 3;'

This can be read as begin notational phrase mode, radical, up 3, base line, 27, end radical, equals 3, end notational phrase mode. The notational phrase is used here because the expression is used within a statement of comparison.

 The expression for the (m+n)th root of (p+q) would be expressed in print as:

and in Braille as: ;>^(m+n)"p+q[

Note that the notational phrase is not used here because the expression is not used within a statement of comparison.

An indexed radical may be preceded by other terms or factors.

**Example:** 3x + y = 3(x+y) ;#3x+y>^3"27[ = 3(x+y)

**Nested (Higher Order) Radicals (Radicals Within Radicals)**

Occasionally we will find that the radicand (expression within a radical) contains another radical. Each nesting, or complexity level, requires special indicators that encompass that part of the complex radical. The method used here is similar to the one used to identify each level of higher-order fractions.

**Nested radical symbols:**

Begin End

Innermost radical > [

Level 2 radical ,> ,[

Level 3 radical ,,> ,,[ (outermost in our examples)

**Examples**

 1 ;,>x+>x+y[+y,[

 2 ;,>^3"x^2"+>^3"x^2"+y^2"[+y^2",[



3 ;;,>>^3"x[,[ = ,>^3">x[,[;'



4 ;,,>x+,>y+>z[,[,,[

**Spatial Representation of Radicals**

It is often helpful to represent radicals spatially. A spatial representation is appropriate when the complexity of the radical expression is such that a spatial representation is clearer than a linear representation.

**Rules regarding the spatial representation of radicals**

1) As with all spatial arrangements, blank lines are required before and after the display. Opening and closing notational phrase indicators will be placed at the beginnings of the required blank lines.

2) In a spatial representation of a higher-order radical expression, that expression must be positioned so that the identifier number (if any) is at the same level as the highest-order radical sign. Since the identifier will fall within the notational phrase, no notational indicator will be required before this identifier, but it may optionally be added.

3) Other than the radical sign and the radicand, there are two structures that provide the expression with its spatiality, namely its ascenders and its vinicula.

a) A vinculum,3 a row of dots 25 that starts in the cell following the top element of an ascender and extends to the cell directly above the last cell of the radicand for that radical.

b) An ascender of dots 56, ; which appears above either a radical sign or above any dot 456 \_ below it; it is at the level of the viniculum.

c) An ascender element (456), \_ which appears above its radical sign and leads to the dot (56) element that is at the level of the viniculum. In many cases this element is not necessary.

4) The order of the radical is not indicated in a spatial representation, since the extent and position of each ascender and vinculum provides this information. (The dot 6 is not used in spatial representations of radicals.)

5) The end radical symbol is not needed because the end of the viniculum for each level clearly shows that end.

**Examples:**

Note the beginning and ending notational phrase indicators placed before and after the braille section of the display below. These have been placed to simulate a page where there were not any print examples.

;#

 ;333

>x+y

 ;33333333

\_ ;333

>x+>x+y+y

 ;3333333333333333333333333

\_ ;33333333333

>^3"x^2"+>^3"x^2"+y^2"+y^2

 ;33333 ;33333

\_;3333 \_ ;3

>>^3"x = >^3">x

;3333333

 \_ ;3333

\_ \_ ;3

>x+>y+>z

;'

**Fractions and Radicals Together in Spatial Representation**

It is often advisable to use a spatial representation for an expression that includes both fractions and radicals. Consider the advantage in readability of the quadratic equation when spatial representation is used.



Linear ;;x = ?-b\_+>b^2"-4ac[/2a#;'

-b\_+>b^2"-4ac[

Spatial ;;x = ?33333333333333#

2a;'

Although the linear form requires fewer braille lines, the spatial form makes it much easier for younger readers to follow the structure of the expression. Note the end notational phrase indicator after the denominator; this is where a computer would need to place it to end the whole phrase.

**Drill #9**

1. The diagonal length across the bedroom is d = .

2. Simplify these radicals:

a. b. c. d. ( )3 e.

3. Find:

a. - b. 2 - 3 c. (7-2) (7+2 ) d.

4. Simplify: a. b.

5. Find:

6. Find: a. + b. ∙ ∙ c.

7. Express each of the following as an integer or as a single radical.

a. b. c.

**Chapter 6 Exercises**

Provide answers in both linear and display formats.

1. 2. 3.

4.

5. (r2)3

6.

7.

8.

9. 4 - 3

7 + 2

10.

11.

12.

13.

14.

15. =

**Chapter 7**

**Geometry Shapes and Modifiers**

**In General**

This chapter on Geometry and Trigonometry, provides the symbols needed for handling geometry and trigonometry as well as necessary rules for correct use of those symbols within a transcription.

**Geometry**

Geometry is the study of the properties and relationships involving lines and planes, etc. It is divided into two basic areas – plane geometry, which deals with two-dimensional figures such as lines and surfaces - and solid geometry which deals with three dimensional figures such as spheres and cubes.

**Shapes in Geometry**

In the study of mathematics, especially in the area of geometry, the representation of shapes is important. Key shapes and their braille form are shown in the table on the following page. This is a complete list except that others may be devised using the basic guidelines shown here; transcriber's notes would be used to explain such additions. These shapes are all notational and each requires a notational indicator.

Shapes are brailled using three braille cells; the first two form the **compound** **prefix** and the third is the **root** which identifies the particular shape.

In the braille column of the table below, note the compound prefix in the first two cells \_" (456,5).

Note the third cell in the Braille column, the root, formed by a letter or symbol.

The three cells together stand for the shape and would be preceded by a notational indicator if it was not already within a notational word or phrase.

Shape Print Braille Dots

------------------------------ --------- ----------- ---------------------

Angle ∠ \_"! (456,5,2346)

Arc - concave ⌣ \_"a (456,5,1)

- convex ⌢ \_"' (456,5,3)

Circle (ring) \_"c (456,5,14)

Diamond \_"d (456,5,145)

Ellipse \_"e (456,5,15)

Equilateral triangle \_"3 (456,5,25)

Hexagon \_"6 (456,5,235)

Is parallel ║ \_"l (456,5,123)

Is perpendicular \_"p (456,5,1234)

Octagon \_"8 (456,5,236)

Oval \_"o (456,5,135)

Parallelogram \_"g (456,5,1245)

Pentagon \_"5 (456,5,26)

Quadrilateral \_"q (456,5,12345)

Rectangle \_"r (456,5,1235)

Rhombus ◊ \_"h (456,5,125)

Right angle \_"? (456,5,1456)

Right triangle \_"v (456,5,1236)

Square \_"4 (456,5,256)

Star \_"s (456,5,234)

Trapezoid \_"z (456,5,1356)

Triangle, generic \_"t (456,5,2345)

Measure of angle m∠ m\_"! (134,456,5,2346)

**Shading of Shapes**

A shape may be shown as hollow, shaded, or solid. The shapes shown in the table above are hollow. A solid shape will be shown with the prefix and then \_ (456) will be inserted before the root. A shaded shape will be shown with the prefix and then **.** (46) will be inserted before the root.

 **Examples:**

solid rectangle \_"\_r (456,5,456,1235)

 shaded square \_"­.4 (456,5,46,256)

Since all of the shapes are notational, the notational indicator is required in expressions such as those above. They will be added any time a shape is used unless the notational mode has already been established for the word or phrase of which the shape is a part.

An exception to the solid rule is the solid dot which is represented by the symbol for a notational period. Solid dot • ] (12456)

**Geometric Shapes in Plane Geometry**

Many of the shapes discussed above are used in the presentation of problems in geometry. When a parallelogram has vertices of M, N, O, and P, it is more convenient to refer to this as MNOP rather than labeling it as “parallelogram MNOP”.

These shape symbols can be used to identify geometric shapes by their vertices; the corners of a shape are usually identified by capital letters. In braille they are individually capitalized*.* If the notational mode has not been established, the shape symbol must be preceded by the notational indicator.

A blank cell is required between the shape and the identifying letters. It is necessary to place a notational indicator before the identifying letters unless the shape is a part of a group of three or more notational words; in that case, the notational phrase indicators will be used.

**Examples:**

Square with vertices W, X, Y, Z WXYZ ;\_"4 ;,w,x,y,z

Rhombus with vertices R, S, T, U RSTU ;\_"h ;,r,s,t,u

Triangle with vertices A, B, C ABC ;\_"t ;,a,b,c

**Examples:**

1. MNO is isosceles.

;\_"t ;,m,n,o is isosceles4

2. Rhombus: CDEF has an area of 50 sq. in.

,rhombusd3 ;\_"h ;,c,d,e,f has an >ea ( #50 sq4 in4

3. m ∠ ABC is 25° indicates that the measure of angle ABC is 25 degrees.

;m\_"! ;,a,b,c is #25@] 9dicates t ! m1sure ( angle ;,a,b,c is #25 degrees4

4. Find the area of ABC.

,f9d ! >ea ( ;\_"t ;,a,b,c]

5. What is the perimeter of MNOP?

,:at is ! p]imet] ( ;\_"4 ;,m,n,o,p\_8

**Modifiers**

Occasionally a math expression will have shapes or text above or below the principal element of the expression. These shapes or text are called

1) modifiers or 2) overscripts or underscripts.

The simplest of these to express in braille are those shapes that can be used as modifiers in the **“modification”** technique.

There are **two groups** of shapes that can be used as modifiers. These groups are “General Modifiers” and “Geometric Modifiers”. The elements of each of these groups are listed below.

**General Modifiers**

acute accent ́ @9 (4,35)

asterisk \* \_[ (456,246)

breve ⌣ \_"a (456,5,1)

circumflex accent ∧ ,5 (6,26)

diaeresis **¨** .3 (46,25)

dot • ] (12456)

grave accent ˋ @5 (4,26)

hat ∧ ,5 (6,26)

left arrow ← @[ (4,246)

macron (bar) − : (156)

right arrow → @o (4,135)

ring ∘ .+ (46,346)

tilde ∼ .9 (46,35)

two-way arrow ↔ @= (4,123456)

umlaut **¨** .3 (46,25)

The general modifiers may be used any place where print text uses them in both literary and mathematical materials.

**Geometry Modifiers**

full line ←––→ @= (4,123456)

half lines

closed left •––––→ \_@o (456,4,135)

closed right ←––––• \_@[ (456,4,246)

open left ○––––→ [.@o](mailto:.@o) (46,4,135)

open right ←––––○ .@[ (46,4,246)

line segments

left closed, right closed •–––––• \_\_: (456,456,156)

left closed, right open •–––––○ \_.: (456,46,156)

left open, right closed ○–––––• .\_: (46,456,156)

left open, right open ○–––––○ ..: (46,46,156)

**Rules for Modification**

Here are three modified expressions where the “over and under” material is contained in the list of “General Modifiers” shown above. The expressions are notational and a notional indicator must be used if that mode has not already been established.



The following steps are required to express these in braille.

1. Enter the notational indicator if that mode has not previously been established.

2. Enter the directly-over or directly-under indicator.

For modification above use < (126) ( Think: gh – go high?)

For modification below use % (146) (Think: sh – sink here?)

3. Enter the term to be modified.

4. Enter the modification symbol. There is no termination symbol; the modifier forms the end of the modified term.

 The expressions above would be brailled as:

;<xy@o ;<,a.+ ;%,m,n:

**Additional Examples:**  (using “general” modifiers from the first list )

1.  ;<x:

(bar over *x --* (notational indicator , if needed, modification above, letter x, bar)

2.  ;<x.9 (tilde over *x* )

**. . . . . .**

3. 3.142857 #3]<142857] (a repeating decimal)



4. ;<<x:+<y::

(The conjugate of, the conjugate of x plus the conjugate of y. Notice how the  and the  are enclosed in an additional set of modification symbols.)

[notational indicator; begin modification above for the long top bar; then modification above for the x followed by the bar symbol to end the modification of x; then the plus sign; then begin modification above for the y followed by the bar symbol to end the modification of y; then the last bar symbol to end the long top bar modification.]

5.  ;<y,5 (hat over *y*)



6. ;%x@o (right arrow under *x*)

**Limits of the Modification Process**

The “modification” process is used only when the element to be place above or below a math expression is composed of symbols that are included in the lists of either the General Modifiers or the Geometry Modifiers. When it is necessary to place other symbols over or under a math expression, a procedure that uses **overscripts** or **underscripts**is required*.* It is important to understand that **all material** that is shown as text above or below a principle expression but is not either a general modifier or a geometry modifier must use the **“overscript or underscript”** method which will be described in Chapter 10.

**Other Uses of Mathematical Shapes**

Occasionally, an author will use a shape to represent a missing element in a mathematical expression. Usually this is presented in a form where the reader is asked to determine the meaning of the shape, from the context of the expression. Notice that the use of spaces depends on what function they serve.

For signs of comparison, spaces will need to be left before and after them. A statement such as this will be placed within a notational phrase so that those spaces will not be delimiters and the notational mode will still be in effect.

For signs of operation, no spaces are present before or after an operator in print. However, in braille, since a blank cell is required both before and after the identifying letters of a braille shape, we must leave a space before and after the shape. We must keep in mind that in print, there would not be any space after the operator which the shape represents.

**Examples:**

1. Write + or - for each shape.

#1] ,write ;+ or ;- = ea\* %ape4

a. 5 2 = 7 ;;a] 5 \_"t 2 = 7;'

or ;a] ;#5 \_"t 2 = 7;'

b. 9 1 = 8 ;;b] 9 \_"4 1 = 8;'

or ;b] ;#9 \_"4 1 = 8;'

c. (a-b) a = -b ;;c] (a-b) \_"4 a = -b;'

or ;c] ;;(a-b) \_"4 a = -b;'

2. 1 qt = pt and 4 qt = ○pt.

;#2] 1 qt = \_"4 pt and 4 qt = \_"c pt];'

or 2] ;#1 qt = \_"4 pt and 4 qt = \_"c pt];'

**Modifiers in Plane Geometry**

The second list of modifiers is used to describe various forms of lines - full lines which extend infinitely, half lines which have an endpoint at one end and extend infinitely in the opposite direction, and line segments which are limited at both ends. Here are some examples of the modification process used in the geometric context.

**Examples:** (of geometric modifiers)

←→ ;<,a,b@= (full-line AB)

AB

○–→ ;<,p,q.@o (The half-line PQ excludes point P.)

PQ

•–––○ ;<,c,d\_.: (The line segment CD in which C is

CD included and D is excluded.)

In the case of half-lines and line segments as shown in print, a solid dot is used at the end of the modifier if the endpoint is included, and a hollow dot is used if the endpoint is excluded. In braille, each of these expressions must be preceded by a notational indicator if the notational mode has not previously been established.

**Drill # 10**

1. What is the area of HIJK if each side is 8 inches long?

2. What is the perimeter of ABCDEF if all sides are 2 feet long?

3. BC ┴ DE

4. MN || OP

5. OP + QR = OR

6. XYZ is isosceles.

7. Find the area of ABC if the base BC = 15 and the height equals 4.

D E

8. Find the perimeter of CDEF where CD=8 and DE=5.

C F

9. ABC is a right triangle with hypotenuse BC.

10. Lines AB and CD intersect at P.

**Chapter 7 Exercises**

1. XY RS

2. AB ⊥ CD

3. 7 = 4

4. ADM = BEM

5. \_ ABD + \_ DBE = ?

6. A B

7. X Y = XY

8. \_ a + \_ b + \_ c = 180o

9. Does equal ?

10. 0.236778 7.1313 …

11. In ABCD, BC = AD. If diagonals AC and BD intersect at E, prove that DEA CEB.

12. AB CD T

**Chapter 8**

**Trigonometry**

Trigonometry is characterized by the generous use of function abbreviations. It is customary for these function abbreviations to be printed in the host type. All of the trigonometric function abbreviations are listed below. The notational indicator is necessary for each abbreviation when it is used in a mathematical context; since they are notational and do not use contractions.

arccosecant arccsc inverse cosine cos-1

arccosine arccos inverse cotangent cot-1

arccotangent arccot inverse cotangent ctn-1

arccotangent arcctn inverse hyperbolic cosecant csch-1

arcsecant arcsec inverse hyperbolic cosine cosh-1

arcsine arcsin inverse hyperbolic cotangent coth-1

arctangent arctan inverse hyperbolic cotangent ctnh-1

cosecant csc inverse hyperbolic secant sech-1

cosine cos inverse hyperbolic sine sinh-1

cotangent cot inverse hyperbolic tangent tanh-1

cotangent ctn inverse secant sec-1

hyperbolic cosecant csch inverse sine sin-1

hyperbolic cosine cosh inverse tangent tan-1

hyperbolic cotangent coth radian rad

hyperbolic cotangent ctnh secant sec

hyperbolic secant sech tangent tan

hyperbolic sine sinh

hyperbolic tangent tanh

There are other function abbreviations which are not specific to trigonometry. They will follow the same basic rules for notation. They will be provided in an appendix.

**Rules for the Use of Trigonometric Functions**

1. If a function name is capitalized in print, it should be shown as such in braille using dot 6. Although in their normal mathematical context, abbreviations are not contracted, if they appear in print in a descriptive passage, contractions may be used.

2. Space must be left between a function abbreviation and its argument. The notational mode must be re-established after this space unless the function is part of a notational phrase in which there is no need for an additional notational indicator.

3. If the function abbreviation carries a subscript, a superscript, an underscript, an overscript, a direct modifier, or any combination of these, such modifications apply to the entire function name. A space is still required *before* its argument.

4. There must also be a space *after* its argument, (after related punctuation, if any).

5. If two or more function abbreviations are consecutive, a notational phrase is used because there will be three or more notational words in the construct.

6. When a function is used as an exponent, it is necessary to re-establish both the notational mode and the superscript level because the required space has ended both. Similarly, the space before an argument will automatically return a superscript or subscript to the baseline so there will be no need for a dot 5 before the argument.

**Examples:**

Note: If the notational mode has not been established, a notational indicator is required before the function name as well as before its argument.

sin x ;sin ;x

cos2  x ;cos^2 ;x

esin x  ;e^sin ;^x

arc AOB ;arc ;,a,o,b

arc sin x ;;arc sin x;' (a space in the function name in print.)

arccosine x ;arccos ;x (no space in the function name in print.)

sin-1  φ = 1/sin φ ( Φ is Greek capital phi \_f)

;;sin^-1 \_f = ?1/sin \_f#;'

**Drill #11**

1. sin (x+y ) 2. sin 

3. sin 30˚ cos 45˚ 4. sin x cos y

5. 2sin x +3cos y 6. Are sine and arcsine true functions?

7. 2 sin 45o cos 45o = sin 90 o

8. cos2 x sin2 x

9. Solve esin x when x = 45° .

10. tan θ =

11. sin 

12. cos 225° = -

13. cos 30° = =

14. Evaluate the expression sin 30˚cos 45˚.

15. Evaluate 2sin x +3cos y if x equals 10˚ and y equals 35˚.

16. What is the meaning of arc cos θ?

**Chapter 9**

**Determinants and Matrices**

**Signs of Grouping**

The list below gives grouping signs defined in NUBS that were not provided in the NUBS Literary Manual.

Symbol Braille Definition Dots listing

{ \_.( left brace (456,46,12356)

} \_.) right brace (456,46,23456)

[ \_@( left bracket (456,4,12356)

] \_@) right bracket (456,4,23456)

**[** @\_( left boldface bracket (4,456,12356)

**]**  @\_) right boldface bracket (4,456,23456)

**|** @\_\ vertical bar left and right (4,456,1256)

|| @@\ double vertical bars left and right (4,4,1256)

.,( extended left brace (46,6,12356)

.,) extended right brace (46,6,23456)

@,( extended left bracket (4,6,12356)

@,) extended right bracket (4,6,23456)

,( extended left parenthesis (6,12356)

,) extended right parenthesis (6,23456)

.,6 extended left transcriber's enclosure (46,235)

.,4 extended right transcriber's enclosure (46,256)

@,\ extended vertical bar left and right (4,46,1256)

@;( lower left half bracket (4,56,12356)

@;) lower right half bracket (4,56,23456)

@^( upper left half bracket (4,45,12356

@^) upper right half bracket (4,45,23456)

**Examples for extended enclosures:**

;#

.,( x+2y = 6.,) ,( x+2y = 8,)

.,(2x-2y = 7.,) ,(2x-3y = 2,)

;'

**Determinants and Matrices**

Determinants and matrices are spatial arrangements, both in print and in braille. They contain arrays of numbers, variables, trigonometric functions, etc. They are readily recognized by their format in print. Here are some print examples of such arrays.

a11 a12 a13

a21  a22  a23

a31 a32 a33

**Rules for Determinants and Matrices**

1. If the array has an identifier label, that label should be aligned with the top line of the array.

2. Notice the treatment of ellipses in arrays, representing missing elements in a single row or an entire missing row or column.

3. Make every effort to braille a matrix or determinant across a single block of lines. In problems involving more than one matrix or determinant, it may be necessary to place the matrices one above the other. This may mean that

a. An identifier must stand alone on a line by itself,

b. A preceding part of the math expression must be separated from the rest of the expression as shown in the third example below or

c. Two determinants that are multiplied together must be separated.

4. Matrices and determinants are always brailled in a spatial format so the notational indicators are needed on the lines above and below them.

**Examples:**

1. sin x cos y

-cos x sin y

;#

1] @,\ sin x cos y@,\

@,\-cos x sin y@,\

;'

2.a11 a12 a13

a21  a22  a23

a31 a32 a33

;#

2] @,(a11 a12 a13@,)

@,(a21 a22 a23@,)

@,(a31 a32 a33@,)

;'

dy1 dy1 . . . dy1

dx1  dx2 dxn

3. dT = . . . . . . . . . .

dyn dyn  . . . dyn

dx1 dx2 dxn

;#

3] d,t

= ,( dy1 dy1 dy1 ,)

,(?333# ?333# ,,' ?3333#,)

,( dx1 dx2 dx;n ,)

,( """""""""""""""""""""" ,)

,( dy;n dy;n dy;n ,)

,(?3333# ?3333# ,,' ?3333#,)

,( dx1 dx2 dx;n ,)

;'

4 . y1  = b11 b12 . . . b1n

y2 b21  b22 . . . b2n

y3 b31 b32  . . . b3n

. . . . . . . . . . . . .

yn br1 br2  . . . brn

;#

4} @,\y1 @,\

@,\y2 @,\

@,\y3 @,\

@,\ ,,' @,\

@,\y4 @,\

@,\y;n @,\

= @,\b11 b12 ,,' b;1n"@,\

@,\b21 b22 ,,' b;2n"@,\

@,\b31 b32 ,, b;3n"@,\

@,\ """"""""""""""""""" @,\

@,\b;r#1 b;r#2 ,,' b;rn"@,\

;'

**Chapter 10 Calculus**

**(Integral and Differential) Summation**

**Logic and Set Theory**

**In General**

The field of calculus is one of the most important areas of mathematical study, especially in the field of engineering. Calculus, which at one time was introduced at the college level, is now being taught to high school juniors and seniors.

**Symbols Used in Calculus**

Many special symbols are needed for expressing mathematical processes involving calculus. Here is a list of the most common calculus symbols. They are all notational.

Name Symbol Braille Dots

------------------------------------ ---------- --------- ------------------------------------

Absolute value **|** @\ (4,1256)

Caret **^** ,5 (6,26)

Del (or nabla) \_j (456,245)

Double vertical bar **| |** @@\ (4,4,1256)

Factorial ! \_! (456,2346)

Infinity @8 (4,236)

Integral ! (2346)

Large boldface pi \_,p (456,6,1234)

Large sigma \_,s (456,6,234)

Limit lim lim (123,24,134)

Logarithm log log (123,135,1245)

Lower limit lim %lim: (146,123,24,134,156)

Natural logarithm ln ln (123,1345)

Norm ║ @@\ (4,4,1256)

Partial derivative @$ (4,1246)

Upper limit lim <lim: (126,123,24,134,156)

Vertical bar **|** @\ (4,1256)

**Basic Use of These Symbols**

The examples below provide some examples of the use of the symbols given above.

Many of them will also be used with underscripts and overscripts; Those will be addressed in the next section.

**Examples:**

(Use of the caret) .37^68 ;]37,568

The symbol " " is called "del" or "nabla".

,! Symbol ;,8\_j,0 is call$ ,8del,0 or ,8nabla,04

The absolute value of -4 is written as │-4│.

,! Absolute value ( #-4 is writt5 z

;@\-4@\]

Sigma " " is often used to mean "summation".

,sigma ;,8\_'s,0 is (t5 us$ 6m1n ,8summa;n,04

<-∞, +∞ > = **R** where **R** represents all real numbers.

;;@<-@8\* +@8@> = \_^,r where \_^,r repres5ts all r1l numb]s];'

(Note: Recall that \_; before a letter makes it a boldface character.)

In this example, the words were spelled out as they must be if they are in the notational mode. We left them in notational mode as a matter of choice. We could have ended the phrase after the first **R,** used contractions and made the second **R** notational separately. Both ways are correct.

6! means 1x2x3x4x5x6. (Six factorial)

#6\_! m1ns #1.[2.[3.[4.[5.[6] \_(Six factorial\_)

**Overscripts and Underscripts**

In addition to subscripts and superscripts, mathematical notation permits stacked notation. In notation of this kind, the principal expression is modified by subsidiary expressions which are written below the principal expression, above the principal expression or both. These subsidiary expressions are called underscripts and overscripts. The principal expression may contain more than one underscript, more than one overscript, or more than one of both. These are written at successively more remote levels relative to the principal expression.

When any expression is accompanied by some print notation above or below the principal expression, such notation must be properly represented in braille. This issue has been addressed to a limited degree by the use of subscripts, superscripts and the process called "modification". This does not cover all of the material that may appear "over" or "under" other expressions. The "modification" technique discussed in Chapter 7 was limited to a specific list of permissible modifiers.

The use of "overscripts and "underscripts" provides a technique for representing all of the "over- under" expressions that occur in scientific texts. This process should be used only when the "modification" process is not applicable. NUBS has a technique for treating such text, which occurs frequently in the use of the symbols for functions in this lesson. This process is especially useful in the proper representation of text that appears above and below the symbols listed in the list of Symbols Used In Calculus.

**Indicators for Overscripts and Underscripts**

Underscripts must be entered first.

First level underscript ;& (56,12346) First level overscript ^& (45,12346)

Intermediate indicator & (12346) to follow each script

Final terminator [ (246) to appear at the end of the final script.

Start the expression with a notational indicator, dots (56) and then the character that will have an underscript and or an overscript. Then the underscript indicator followed by the information in the underscript is entered. If there is an overscript to be entered, the intermediate terminator is placed after the underscript. The overscript indicator followed by the overscript is entered and then the final terminator is entered. If there was not to be an overscript, the final terminator rather than the intermediate indicator would have been used after the underscript.

If there are more than one level of underscript or overscript, additional ;, dots (56) or ^, dots (45) indicators would be added for each subsequent level of underscript, or overscript. Whether there is one or there are several such prefixes, each sequence must be followed by the & (12346) character which is called the intermediate indicator. The intermediate indicator is used after each level of script that is not the last script on the principal expression. The final terminator follows the last script being used on any particular principal expression to show that the end of the stack has been reached.

Keep in mind that when expressions are used as functions, a space is required before its argument so notational phrase indicators are required. Thus, some of the examples below will start with two notational indicators, dots (56,56) and end with dots (56,3).

When there are signs of comparison within a script, the usual spaces around that sign will be omitted.

**Examples of underscripts and overscripts** using some of the symbols from the list of symbols used in calculus.

;;lim;&x@o0[ f(x);'

;;lim;&x@oa^+"[ f(x);'

Note in this example that the plus sign after the letter "a" is part of the underscript as shown by the fact that the underscript has not yet been terminated.

Σ xn  ;;\_,s ;& **n=0&^&@8[ x;n;'**

n =0

n

Σ ai j  ;;\_,s ;& **i\*j&;;&i\_<j\_<n&^&n[ a;ij;'**

i, j

i ≤ j ≤ n

**The Integral Sign**

As its name implies, the integral sign represents the process of integrating a function over a specified region. The integral sign will often carry numbers that look like superscripts and subscripts to define the limit over which the integration applies. As with other functions, a space must be placed before the argument so notational phrase notation is required.

*dx* means to find the area under the curve for y = x2 that lies between 0 and 4. The 0 and the 4 are called the limits of the integration. The expression above would be brailled as:

;;!;0^4 x^2 dx;'

which could be read as: Integral, sub 0, sup 4, baseline, x, sup 2, baseline, dx

Sometimes signs will have a shape superimposed on them. When these occur, in braille, the sign should be followed by the overstrike indicator, (5,12346) "& which then is followed by the superimposed shape.

**Examples of superimposed signs**

;!"&\_"c Integral sign with circle for overstrike

Notational Ind.; Integral ! overstrike indicator "& circle symbol \_"c

( xn );(x;n")"&\_/ Expression with a slash overstrike

Sometimes you will encounter integral signs as a pair or a triplet. In such cases, the integral sign is simply repeated as indicated.

;!! ;!!!

Here are some examples of expressions involving integrals.

*dx*  ;;! e^-x^^2 dx;'

*dx* = x2| = (8-0) =

;;!;0^2 x^2 dx = ?1/3#x^2 @\;0^2

= ?1/3#(8-0) = ?8/3#;'

;;!;1^2 !;0^x-1 y dydx;'

∫∫(x2+y2)dxdy ;;!!;&,d[ (x^2"+y^2")dxdy;'

D

**Logic and Set Theory**

Logic and set theory are closely related branches of mathematics and thus share many symbols. Most of those symbols can be classified either as operation signs or as comparison signs, and have been included in their respective categories in the Mathematics Symbol Set together with their tactile graphics. They are governed by the same rules that we have already encountered regarding other operation signs and comparison signs.

Note that the symbols for "meet" and "join" are top-to-bottom symmetric in print; they are likewise top-to-bottom symmetric in NUBS. The same observation holds for the signs for "union" and "intersection."

Note that the prefix . (46) denotes negation. It is placed either before the first character of the principal sign to achieve negation, or it replaces that first character.

The comparison signs in the Logic and Set Theory list have been duplicated in the Signs of Comparison list, and the operation signs in the Logic and Set Theory list have been duplicated in the Operation Signs list. Four entries in the Logic and Set Theory list are neither comparison signs nor operation signs.

**Logic and Set Theory Characters**

Cartesian product .[ (46,246) **×**

contains the element @@i (4,4,24) ∈

contains the subset @@y (4,4,13456) 

does not contain the element .@i (46,4,24) ∉

does not contain the subset .@y (46,4,13456) 

empty set \_0 (456,356) ∅

for all \_\_' (456,456,3) ∀

global Cartesian product \_.[ (456,46,246) **×**

global intersection \_@m (456,4,134)  **⊂**

global product \_,p (456,6,1234)

global sum \_,s (456,6,234)

global union \_@u (456,4,136)  **∪**

if and only if @= (4,123456) iff

implies @@y (4,4,13456) ⊃

intersection @m (4,134) ⊂

is an element of @@e (4,4,15) ∈

is implied by @@& (4,4,12346) ∩

is not an element of .@e (46,4,15) ∉

is not a subset of .@& (46,4,12346) ⊄

is a proper subset of ­\_@& (456,4,12346) ⊂

is a subset of @@& (4,4,12346) ⊆

join @v (4,1236) ∧

meet @p (4,1234) ∨

there exists \_\_5 (456,456,26) ∃

there exists uniquely \_.5 (456,46,26) ∃⏐

union @u (4,136) ∪

**Appendix MA**

**Signs of Operation**

minus - - (36)

plus + + (346)

multiplication asterisk \* \_[ (456,246)

multiplication times cross × .[ (46,246)

multiplication times dot " ,] (6,12456)

divided by ./ (46,34)

slash \_/ (456,34)

begin displayed division o (135)

**Appendix MB**

**Signs of Comparison**

is equal to = (123456) =

is greater than @> (4,345) >

is less than @< (4,126) <

is not equal to .= (46,123456) ≠

approximately equal \_9 (456,35) ≈

contains the element @@i (4,4,24) 

contains the subset @@Y (4,4,13456) 

does not contain

the element .@I (46,4,24) ∉

does not include .@Y (46,4,13456) 

greater than or equal \_> (456,345) 

implies @@y (4,4,13456) 

includes @@y (4,4,13456) 

is an element @@e (4,4,15) 

is a subset @@& (4,4,12346) 

is not an element .@E (46,4,15) ∉

is not a subset .@& (46,4,12346) 

less than or equal \_< (456,126) 

not equal to .= (46,123456) 

not greater than .> (46,345) 

not less than .< (46,126) 

**Appendix MC**

**Fractions and Radicals**

begin fraction ? (1456)

fraction bar / (34) (Not a two-cell slash !)

end fraction # (3456)

**Symbols used in Cancellation**

Begin cancellation & (12346)

End cancellation [ (246)

One-character cancellation \_ (456)

**Symbols used in complex fractions:**

Open complex fraction ,? (6,1456)

Complex fraction line ,/ (6,34)

Close complex fraction ,# (6,3456)

Begin outermost fraction ,,? (6,6,1456)

Outermost fraction bar ,,/ (6,6,34)

End outermost fraction ,,# (6,6,3456)

**Symbols used in Radicals (Square Root)**

Begin radical > (345)

End radical [ (246)

**Symbols used in Nested radicals:**

Complexity indicator for

fractions and radicals , (6)

Begin End

Innermost radical > [

Level 2 radical ,> ,[

Level 3 radical ,,> ,,[

Component of a radical ascender \_ (456)

Component of spatial fraction bar 3 (25)

Component of a spatial vinculum 3 (25)

Last component of radical ascender ; (56)

Radical sign without vinculum ..> (46,46,345)

**Appendix MD**

**Symbols used for Geometry**

The three cells together stand for the shape.

Shape Print Braille Dots

------------------------------ --------- ----------- ---------------------

Angle ∠ \_"! (456,5,2346)

Arc - concave ⌣ \_"a (456,5,1)

- convex ⌢ \_"' (456,5,3)

Circle (ring) \_"c (456,5,14)

Diamond \_"d (456,5,145)

Ellipse \_"e (456,5,15)

Equilateral triangle \_"3 (456,5,25)

Hexagon \_"6 (456,5,235)

Is parallel ║ \_"l (456,5,123)

Is perpendicular \_"p (456,5,1234)

Octagon \_"8 (456,5,236)

Oval \_"o (456,5,135)

Parallelogram \_"g (456,5,1245)

Pentagon \_"5 (456,5,26)

Quadrilateral \_"q (456,5,12345)

Rectangle \_"r (456,5,1235)

Rhombus ◊ \_"h (456,5,125)

Right angle \_"? (456,5,1456)

Right triangle \_"v (456,5,1236)

Square \_"4 (456,5,256)

Star \_"s (456,5,234)

Trapezoid \_"z (456,5,1356)

Triangle, generic \_"t (456,5,2345)

Measure of angle m∠ m\_"! (134,456,5,2346)

Adding dots (456) between the dot (5) and the shape designating letter makes the shape a solid filled in shape.

Adding dots (46) between the dot (5) and the shape designating letter makes the shape a shaded in shape.

**Examples:** solid rectangle \_"\_r (456,5,456,1235)

shaded square \_"­.4 (456,5,46,256)

An exception to the solid rule is the solid dot which is represented by the symbol for notational period. Solid dot • ] (12456)

**General Modifiers**

acute accent ́ @9 (4,35)

asterisk \* \_[ (456,246)

breve ⌣ \_"a (456,5,1)

circumflex accent ∧ ,5 (6,26)

diaeresis **¨** .3 (46,25)

dot • ] (12456)

grave accent ˋ @5 (4,26)

hat ∧ ,5 (6,26)

left arrow ← @[ (4,246)

macron (bar) − : (156)

right arrow → @o (4,135)

ring ∘ .+ (46,346)

tilde ∼ .9 (46,35)

two-way arrow ↔ @= (4,123456)

umlaut **¨** .3 (46,25)

**Geometry Modifiers**

full line ←––→ @= (4,123456)

half lines

closed left •––––→ \_@o (456,4,135)

closed right ←––––• \_@[ (456,4,246)

open left ○––––→ [.@o](mailto:.@o) (46,4,135)

open right ←––––○ .@[ (46,4,246)

line segments

left closed, right closed •–––––• \_\_: (456,456,156)

left closed, right open •–––––○ \_.: (456,46,156)

left open, right closed ○–––––• .\_: (46,456,156)

left open, right open ○–––––○ ..: (46,46,156)

**Appendix ME.**

**Trigonometry Functions and Abbreviations**

amplitude amp

antilogarithm antilog

arc arc

argument arg

cologarithm colog

cosine + *i*sin cis

coversine covers

curl curl

determinant det

dimension dim

divergence div

error function erf

exponential exp

exsecant exsec

gradient grad

haversine hav

imaginary im

infimum inf

limit lim

logarithm log

lower limit lim %lim:

maximum max

minimum min

modulo mod

modulus mod

natural logarithm ln

rank rnk

real re

supremum sup

upper limit  <lim:

versine vers

arccosecant arccsc

arccosine arccos

arccotangent arccot

arccotangent arcctn

arcsecant arcsec

arcsine arcsin

arctangent arctan

cosecant csc

cosine cos

cotangent cot

cotangent ctn

cosine cos

cotangent cot

cotangent ctn

hyperbolic cosecant csch

hyperbolic cosine cosh

hyperbolic cotangent coth

hyperbolic cotangent ctnh

hyperbolic secant sech

hyperbolic sine sinh

hyperbolic tangent tanh

inverse cosecant csc-1

inverse cosine cos-1

inverse cotangent cot-1

inverse cotangent ctn-1

inverse hyperbolic cosecant csch-1

inverse hyperbolic cosine cosh-1

inverse hyperbolic cotangent coth-1

inverse hyperbolic cotangent ctnh-1

inverse hyperbolic secant sech-1

inverse hyperbolic sine sinh-1

inverse hyperbolic tangent tanh-1

inverse secant sec-1

inverse sine sin-1

inverse tangent tan-1

radian rad

secant sec

sine sin

tangent tan

**Appendix MF.**

**Signs of Grouping**

**Angle brackets**

left angle bracket @< (4,126) <

right angle bracket @> (4,345) >

**Barred signs**

braces

left .( (46,12356) {

right .) (46,23456) }

brackets

left @( (4,12356) [

right @) (4,23456) ]

**Parentheses**

right ) (6,12356)

left ( (6,23456)

**Transcriber's enclosures**

left .6 (46,235)

right .4 (46,256)

**Vertical bar**

left and right @\ (4,1256)

**Boldface signs**

brackets

left @\_( (4,456,12356) **[**

right @\_) (4,456,23456) **]**

vertical bar

left and right @\_\ (4,456,1256) **|**

**Double vertical bars**

left and right @@\ (4,4,1256) ||

**Extended signs**

braces

left .,( (46,6,12356)

right .,) (46,6,23456)

brackets

left @,( (4,6,12356)

right @,) (4,6,23456)

parentheses

right ,) (6,12356)

left ,( (6,23456)

**Half-brackets**

lower left @;( (4,56,12356)

lower right @;) (4,56,23456)

upper left @^( (4,45,12356)

upper right @^) (4,45,23456)

**Appendix MG.**

**Calculus Symbols**

absolute value @\ (4,1256) |

del \_j (456,245) 

double vertical bar @@\ (4,4,1256) | |

infinity @8 (4,236) ∞

integral ! (2346) ∫

large boldface pi \_,p (456,6,1234) π

large boldface sigma \_,S (456,6,234) ∑

limit lim lim

logarithm log log

lower limit LIminf 

nabla \_j (456,245) 

natural logarithm ln ln

norm @@\ (4,4,1256) ||

partial derivative @$ (4,1246) ∂

prime ' (3) ´

upper limit limsup 

vertical bar @\ (4,1256) **|**

Weierstrass *p* ."p (46,5,1234)

**Appendix MH**

**Greek Alphabet**

**Lower Case Upper Case**

Symbol Name Braille Symbol Name Braille

α alpha .a Α alpha \_a

β beta .b Β beta \_b

γ gamma .g Γ gamma \_g

δ delta .d Δ delta \_d

ε epsilon .e Ε epsilon \_e

ζ zeta .! Ζ zeta \_!

η eta .: Η eta \_:

θ theta .? Θ theta ­ \_?

ι iota .i Ι iota \_i

κ kappa .k Κ kappa \_k

λ lambda .l Λ lambda \_l

μ mu .m Μ mu \_m

ν nu .n Ν nu \_n

ξ xi .x Ξ xi \_x

ο omicron .o Ο omicron \_o

π pi .p Π pi \_p

ρ rho .r Ρ rho \_r

σ sigma .s Σ sigma \_s

τ tau .t Τ tau \_t

υ upsilon .u Υ upsilon ­\_u

φ phi .f Φ phi \_f

χ chi .& Χ chi \_&

ψ psi .y Ψ psi \_y

ω omega .w Ω omega ­ \_w

**Appendix MI.**

**Logic and Set Theory**

Cartesian product .[ (46,246) **×**

contains the element @@i (4,4,24) ∈

contains the subset @@y (4,4,13456) 

does not contain the element .@i (46,4,24) ∉

does not contain the subset .@y (46,4,13456) 

empty set \_0 (456,356) ∅

for all \_\_' (456,456,3) ∀

global Cartesian product \_.[ (456,46,246) **×**

global intersection \_@m (456,4,134)  **⊂**

global product \_,p (456,6,1234)

global sum \_,s (456,6,234)

global union \_@u (456,4,136)  **∪**

if and only if @= (4,123456) iff

implies @@y (4,4,13456) ⊃

intersection @m (4,134) ⊂

is an element of @@e (4,4,15) ∈

is implied by @@& (4,4,12346) ∩

is not an element of .@e (46,4,15) ∉

is not a subset of .@& (46,4,12346) ⊄

is a proper subset of ­\_@& (456,4,12346) ⊂

is a subset of @@& (4,4,12346) ⊆

join @v (4,1236) ∧

meet @p (4,1234) ∨

there exists \_\_5 (456,456,26) ∃

there exists uniquely \_.5 (456,46,26) ∃⏐

union @u (4,136) ∪

**Appendix MJ.**

**Miscellaneous Symbols**

**Arrows**

northeast @^o (4,45,135) 

northwest @^[ (4,45,246) 

northwest-southeast @;= (4,56,123456) 

southeast @;o (4,56,135) 

southwest @;[ (4,56,246) 

southwest-northeast @^= (4,45,123456) 

ampersand @& (4,12346) &

apostrophe ' (3) '

asterisk \_[ (456,246) \*

at sign @a (4,1) @

backslash \_\* (456,16) **\**

caret ,5 (6,26) **^**

colon 3 (25)  **:**

comma \* (16) **,**

crosshatch .# (46,3456) #

dollar sign @s (4,234) $

equals = (123456) =

exclamation 6 (235) !

grave accent @5 (4,26) ´

hyphen - (36) -

percent sign .0 (46,356) %

period ] (12456) **.**

plus + (346) +

question mark 8 (236) ?

quote ,7 (6,2356) “

semicolon 2 (23) **;**

slash \_/ (456,34) /

space (no dots)

tilde .9 (46,35) ~

underscore ,- (6,36) \_

vertical bar @\ (4,1256) |

begin upper-case passage ,,; (6,6,56)

end upper-case passage ,' (6,3)

upper case, one letter , (6)

upper case, one word ,, (6,6)

no boundary @ (4)

overscript ^& (45,12346)

overstrike "& (5,12346)

runover \_ (456)

shaded shape . (46)

solid shape \_ (456)

underscript ;& (56,12346)

**Computer and Calculator**

begin keytop legend [ (246)

begin screen display & (12346)

runover indicator \_ (456)

**Punctuation**

ellipsis, vertical \_1 (456,2)

**Special Letters**

**Crossed**

b @b (4,12)

d @d (4,145)

h @h (4,125)

λ (lambda) ."l (46,5,123)

**Global Operation Signs**

global product \_,p (456,6,1234) П

global sum \_,s (456,6,234) Σ

**Hebrew**

aleph .1 (46,2) אּ

**Number Sets**

complex \_\_c (456,456,14)

integers \_\_i (456,456,24)

natural \_\_n (456,456,1345)

rational \_\_z (456,456,1356)

real \_\_r (456,456,1235)

**Obsolete Greek**

koph or koppa ."q (46,5,12345)

sampi .": (46,5,156)

**Variant Greek**

alpha ."a (46,5,1)

beta ."b (46,5,12)

theta ."? (46,5,1456)

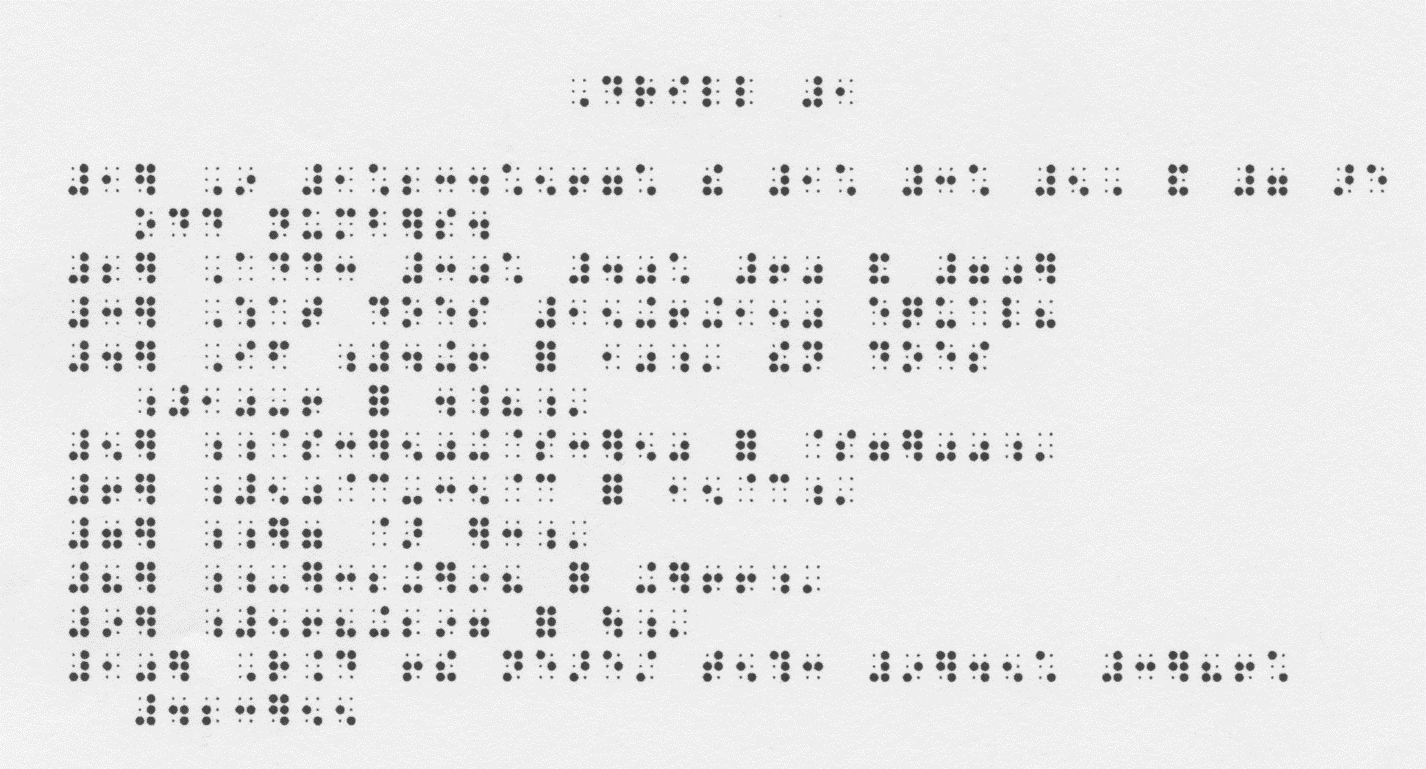
sigma ."s (46,5,234)

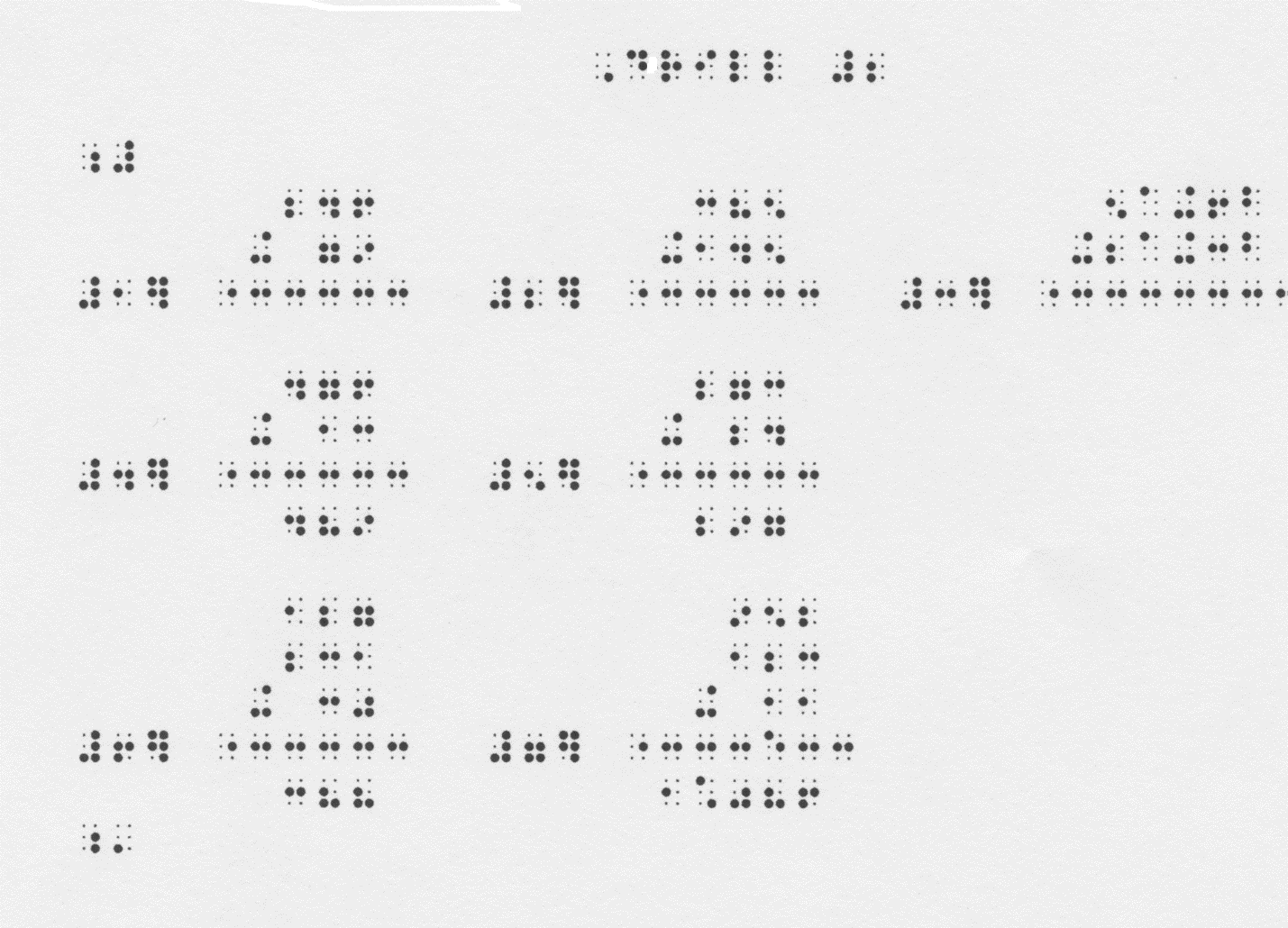
phi ."f (46,5,124)

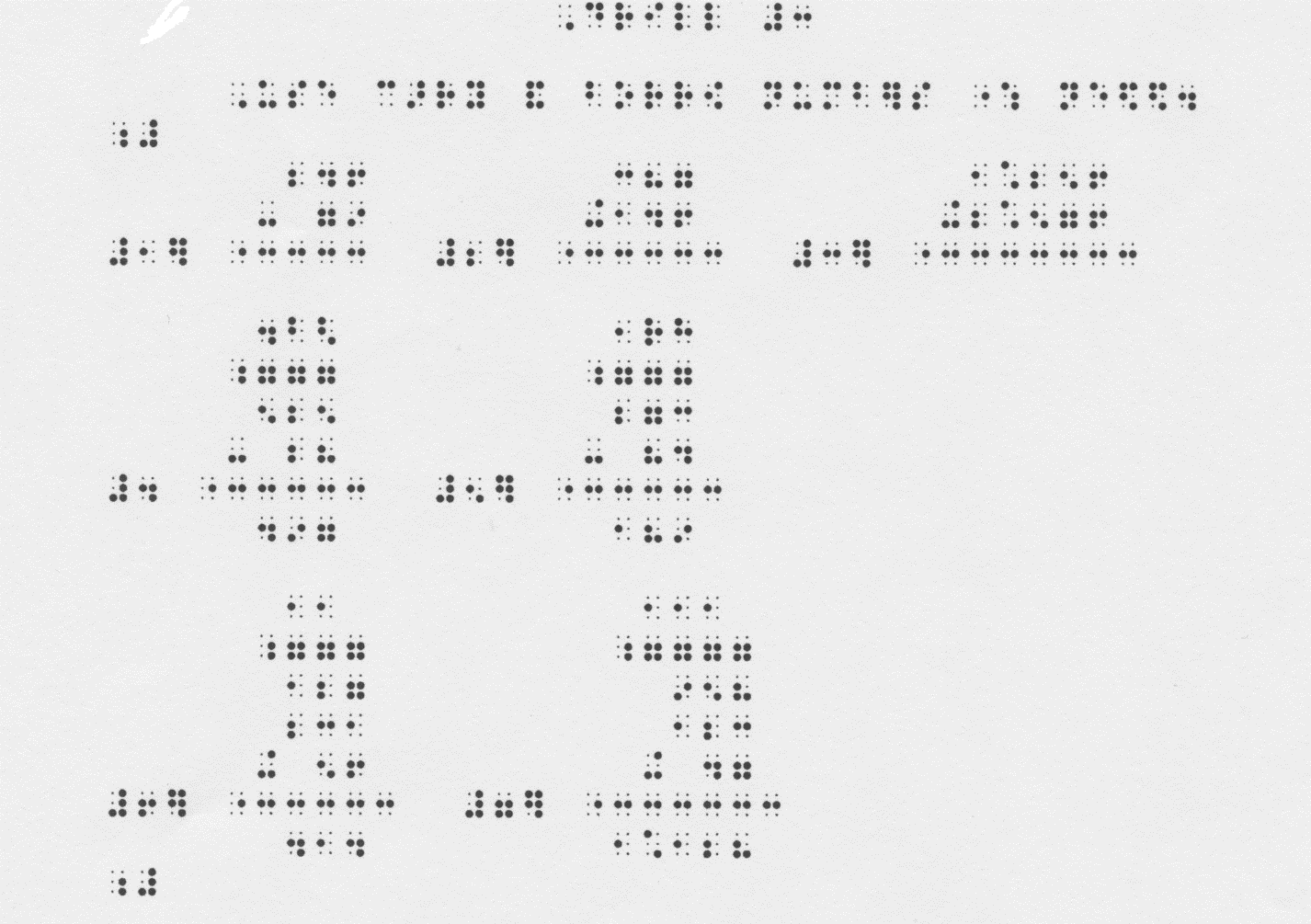
**Appendix MK.**

**Braille Answers for Drills**

**Drill #1**



**Drill #2**

**Drill #3** 

**Drill #4**

,drill #4

;#1] 12.[3.[10 = 360;'

;#2] 25,]3 = \

#3] ,True or false4 ;#33\_[3 @< 100;'

101

101

;777777

326 189 746

.[35 .[31 .[212

4] "3333333 5] "333333 6] "33333333

1 630 189 1 492

9 78 5 67 7 46

"3333333 "333333 149 2

11\*410 5\*859 "33333333

158\*152

2]38 78]2 2837

.[1]7 .[65]3 .[416

7] "333333 8] "33333333 9] "3333333333

1 666 23 46 17 022

2 38 391 0 28 37

"333333 4692 1 134 8

4]046 "33333333 "3333333333

5106]46 1\*180\*192

;'

**Drill #5**

,drill #5

;#1] 1755./15 = 117;'

#2] ,>at is #21./7\_8

;#3] 112./7 = \;'

;#4] 985./\ = 197;'

;#5] \./21 = 13;'

;#6] 36.[4./6 = 24;'

;#

19 32 8 ,r#4

"3333 "3333 "333333333

7] 23o437 8] 18o576 9] 79o636

23 54 632

"3333 "3333 "333333333

207 36 4

207 36

"3333 "3333

0 0

7]47 1 3 ]7

"3333333 "33333333

10] 65o485]55 11] 2]4,5o32]8,58

455 24

"3333333 "33333333

30 5 8 8

26 0 7 2

"3333333 "33333333

4 55 1 6 8

4 55 1 6 8

"3333333 "33333333

0 0

;'

Example #12 shows a problem with only one ruled line. This approach especially shows the need to keep numbers in columns of equal place value.

;#

193 ,r#15

"333333333333

12] 64o12367

64

596

576

207

192

15

;'

**Drill #6**

#1] ;?3/4#

#2] ;?a+b/c+d#

#3] ;;?1/4#+?3/4#-?1/2# = ?1/2#;'

#4] ;;?a/b#.[?c/d# = ?ac/bd#;'

#5] ;(?3/2#a+?1/2#b)

#6] ;;@s3]00.[?1/5# = @s]60;' (Note Dollar sign (4, 234) )

#7] ;;?1\\/1000# = ?1/10#;' (Note Omission signs (1256) )

;#

#3 #9 X+y ;a

8] ?33# 9] ?333# 10] ?333# 11] ?333#

#8 #16 Z b+c

#7 #11 2x+4y

12] 26?333# 13] 7?333# 14] ?33333#

#16 #5 6z

;j

15] 12y?33#

k

;'

**Drill #7**

**#1] ;,??1/4#,/?3/4#,#**

**#2] ;,??3-1/4+3#,/?2-1/3+5#,#**

**#3] ;,?2?1/3#,/4?1/5#,#**

**#4] ,?(?3/2#).[(?1/2#).[(-?1/2#),/1.[2.[3,#**

**#5] ,??x-3/x+1#,/?x/y#,#**

**#6] ,?a+?2/a+1#,/a-?3/a-2#,#**

**;#**

**#1 #3-1 #1**

**#33# ?3333# 2?33#**

**#4 4+3 #3**

**1] ?3333# 2] ?333333# 3] ?3333333#**

**#3 #2-1 #1**

**?33# ?3333# 4?33#**

**#4 3+5 #5**

**;,**

**;#**

**(?3/2).[(?1/2#.[(-?1/2#)**

**#4] ?333333333333333333333333#**

**1.[2.[3**

**;,**

**;#**

**x-3 #2**

**?333# a+?333#**

**X+1 a+1**

**#5] ?33333# #6] ?3333333#**

**x #3**

**?3# a-?333#**

**Y a-2**

**;,**

**;#**

**& [ 3x & [ 6x-9**

**#7] (x-1)(?333#)+(x-1)(?3333#) = (x-1)9**

**x-1 x-1**

**& [ & [**

**;,**

**Drill #8**

#1] ,! ,py?agor1n ,!orem is ;;a^2"+b^2 = c^2";'

#2] ,e9/e9's fam\s equa;n is ;;,e = mc^2"]

#3] ;;y = 9+x^4"-6x^3"+18x+3x^2";'

#4] ;,n,h4 is ammonium4

#5] ;,c6,h12,o6 is glucose4

#6] ;,c12,h22,o11 is sucrose4

#7] ,evaluate3

;#

2\_[8^2"-2^2"\_{8

?333333333333333#

2\_[8

;'

#8] ,copy"r ;@.c #2008 0,! ,mc,graw-,hill ,-panies

#9] ,evaluate if ;;r = 2\* s = 3\* and t = 11\*

s^2"+(r^3"-8)+5\* 3r(r+s)^2"-1\* and

t^2"+8st+r^2";'

**Drill #9**

#1] ,! Diagonal l5g? acr ! b$room is

;;d = >l^2"+w^2"[];'

#2] 'simplify ^! Radicals3

;a] ;>63[

;b] ;>27[

;c] ;>?a+b/a-b#[

;d] ;(r^2">r^3"[)^3

;e] ;>80a^6"b^2"[

#3] ,f9d3

;a] ;>75[->48[

;b] #2>108[-3>27[

;c] ;(7-2>5[)(7+2>5[)

;d] ;>2[>10[

#4] ,simplify3

;a] ;?4>6[-3>2[/7>3[+2>5#[

;b] ;,?2->?1/4#[,/3->?1/2#[,#

#5] ,f9d3 ;>(30)[.[>(10)[.[>(10)[

#6] ,f9d3

;a] ;>^3"16[+>^4"162[

;b] ;>^5"8[,]>^5"2[,]>^5"2[

;c] ;>^4"8r^2"s^3"t^5"[>^4"4r^6"st^2"[

#7] ,express ea\* (! Foll[+ z an 9teg] or z a s+le radical]

;a] ;,>>a[,[

;b] ;,>^3">729[,[

;c] ,,>,>^3">^4"b^2"[,[,,[

**Drill #10**

#1] ,:at is ! >ea ( ;\_"4 ;,h,I,j,k if ea\* side is #8 9\*es l;g8

#2] ,:at is ! p]imet] ( ;\_"6 ;,a,b,c,d,e,f if all sides >e #2 feet l;g8

#3] ;;,b,c \_"p ,d,e;'

#4] ;;,m,n \_"l ,o,p;'

#5] ;;<,o,p:+<,q,r: = <,o,r:;'

#6] ;\_"t ;,x,y,z is isosceles4

#7] ,f9d ! >ea ( ;\_"t ;,a,b,c if ! base

;;<,b,c: = 15;' & ! hei<t equals #4}

#8] ,f9d ! p]imet] ( ;\_"g ;,c,d,e,f ":

;;<,c,d: = 8 and <,d,e: = 5];'

#9] ;\_"v ;a,b,c is a "r triangle ) hypot5use ;,b,c]

#10] ,l9es ;,a,b & ;,c,d 9t]sect at ;,p]

**Drill #11**

#1] ;sin ;(x+y)

#2] ;sin ;.p\_/3

#3] ;;sin 30@] cos 45@];'

#4] ;;sin x cos y;'

#5] ;#2sin x +3cos y;'

#6] ,>e s9e & cos9e true func;ns8

#7] ;#2sin 45@] cos 45@] = sin 90@];'

#8] ;;cos^2 x sin^2 x;'

#9] ,solve ;e^sin ;^x :5 ;;x = 45@]];'

#10] ;;tan .? = ?sin .? /cos .?#;'

#11] ;;sin ?5\_p/12#;'

#12] ;;cos 225@] = ->?1+cos 450@] /2#[;'

#13] ;;cos 30@] = ?x/r# = ?1/2#>3[;'

#14] ,evaluate ! expres.n ;;sin 30@] cos 45@]];'

#15] ,evaluate ;#2sin x +3cos y;' if

;x equals #10@] & ;y equals #35@]]

#16] ,<at is ! m"n+ ( ;;arc cos .?\_8;'